

Essays in Macroeconomics and Finance
Pablo Ottonello

Submitted in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy
in the Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY

2015

© 2015

Pablo Ottonello

All rights reserved

ABSTRACT

Essays in Macroeconomics and Finance

Pablo Ottonello

This dissertation contains three essays on Macroeconomics and Finance. The first chapter has been motivated by the fact that recoveries from financial crises are characterized by low investment rates and declines in capital stocks. The paper constructs an equilibrium framework in which financial shocks have a persistent effect on aggregate investment. The key assumption is that physical capital is traded in a decentralized market with search frictions, generating “capital unemployment.” After a negative financial shock, the share of unemployed capital is high, and the economy dedicates more resources to absorbing existing unemployed capital into production, and less to accumulating new capital. An estimation of the model for the U.S. economy using Bayesian techniques shows that the model can generate the investment persistence and half of the output persistence observed in the Great Recession. Investment search frictions also lead to a different interpretation of the sources of business-cycle fluctuations, with a larger role for financial shocks, which account for 33 percent of output fluctuations. Extending the model to allow for heterogeneity in match productivity, the framework also provides a mechanism for procyclical capital reallocation, as observed in the data.

The second and third chapters focus on labor unemployment during financial crises. The second chapter uses a sample of 116 recession episodes in developed and emerging market economies to compare the labor-market recovery during financial crises with that of other recession episodes. It documents two new stylized facts. First, labor-market recovery from financial crises is characterized by either higher unemployment (“jobless recovery”) or a lower real wage (“wageless recovery”). Second, inflation determines the type of recovery: low inflation (below 30 percent annual rate) is associated with jobless recovery, while high inflation is associated with wage-

less recovery. The paper shows that this pattern of labor recovery from financial crises is consistent with a simple model in which collateral requirements are higher (lower) when a larger share of labor costs (physical capital expenditure) is involved in a loan contract.

The third chapter paper conducts a quantitative study of the optimal exchange-rate policy in a small open economy that faces the “credit access–unemployment” trade-off: In the presence of nominal wage rigidity, exchange-rate depreciation reduces unemployment; in the presence of collateral constraints linking external debt to the value of income, exchange-rate depreciation tightens the collateral constraint and leads to higher consumption adjustment. It is shown that the optimal policy during financial crises generally features large currency depreciation, since welfare costs related to higher unemployment and lower consumption typically outweigh welfare costs associated with intertemporal misallocation of consumption. The optimal policy also implies a lower currency depreciation than that necessary to achieve full employment, which is consistent with a managed-floating exchange-rate policy, frequently observed during financial crises in emerging market economies. Sudden stops (or large current-account adjustments) are part of the endogenous response to large negative shocks under the optimal exchange-rate policy.

Contents

List of Figures	iv
List of Tables	vi
Acknowledgements	ix
1 Capital Unemployment, Financial Shocks, and Investment Slumps	1
1.1 Introduction	1
1.2 Relationship with the Literature	6
1.3 Investment Search Frictions: Basic Framework	12
1.4 A Quantitative Business-Cycle Model with Investment Search Frictions	26
1.5 Quantitative Analysis	44
1.6 Capital Reallocation	56
1.7 Conclusion and Future Research	65
2 Labor Market, Financial Crises and Inflation: Jobless and Wageless Recoveries	69
2.1 Introduction	69
2.2 Data and Descriptive Statistics	73
2.3 Econometric Analysis	80
2.4 Robustness	83
2.5 Financial Crises and Jobless Recoveries: A Simple Analytical Framework	92
2.6 Conclusions	99

3	Optimal Exchange-Rate Policy Under Collateral Constraints and Wage Rigidity	101
3.1	Introduction	101
3.2	The Model Economy	106
3.3	Exchange-Rate Regimes: Definitions and Analytical Results	113
3.4	Quantitative Analysis	122
3.5	Sensitivity Analysis and Extensions	143
3.6	Conclusions	157
	Bibliography	161
A	Appendix for Chapter 1	175
A.1	Data Appendix	175
A.2	Additional Figures	180
A.3	Mapping from Investment Search Frictions to Wedges	185
A.4	Proofs	188
A.5	Benchmark Business Cycle Economy	189
B	Appendix for Chapter 2	193
B.1	List of Recession Episodes	193
B.2	Proofs	195
B.3	A Quantitative Exercise of the Analytical Framework	195
C	Appendix for Chapter 3	199
C.1	Omitted Constraints in Ramsey Problems	199
C.2	Proofs	200
C.3	Data Appendix	202

C.4 Recursive Formulation of the Optimal Exchange-Rate Policy without	
Capital-Control Taxes	203

List of Figures

1.1	Financial Crises and Investment Slumps.	3
1.2	U.S. Unemployment of Physical Capital and Labor, 1980–2013.	5
1.3	A Classification of Capital Stock Status.	10
1.4	Policy Functions.	24
1.5	Transitional Dynamics and Initial Capital-Unemployment Rate.	25
1.6	Agents and Markets.	28
1.7	Structure of Capital Markets, Quantitative Model.	30
1.8	U.S. Great Recession: Predicted Recovery.	53
1.9	Impulse-Responses to Contractionary Shocks.	56
1.10	Structure of Capital Markets, Model with Capital Reallocation.	59
1.11	Impulse Responses to Contractionary Shocks.	63
1.12	Impulse Responses to Contractionary Shocks.	65
2.1	Jobless recovery during the Great Recession	70
2.2	Labor Market Recovery, Financial Crises and Inflation	71
2.3	Inflation in Recession Episodes	76
2.4	House Prices and Financial Crises: Developed Market Economies	92
2.5	Optimal Input Vector under Credit Constraint	96
3.1	Policy Functions.	128
3.2	Financial Crises – Exogenous Variables.	129
3.3	Financial Crises – Endogenous Variables.	131

3.4	Exchange-Rate Policy in Emerging Market Economies– Lehman Episode.	132
3.5	Welfare Costs by State.	135
3.6	Welfare Costs During Financial Crises.	137
3.7	Exchange-Rate Regimes in Argentina and Emerging Market Economies. .	139
3.8	Financial Crises – Model and Data.	140
3.9	Sensitivity of Optimal Policy during Financial Crises.	144
3.10	Financial Crises – Optimal Policy under Alternative Stochastic Structures.	149
3.11	Financial Crises – Optimal Policy under Alternative Model Structures. .	154
3.12	Financial Crises – Optimal Policy with No Capital-Control Taxes.	158
A.1	Unemployment of Capital and Labor, Euro Economies, 2007–2013. . . .	180
A.2	Unemployment of Capital and Utilization, U.S. Recession Episodes. . . .	181
A.3	Impulse-Responses to a Neutral-Technology Shock.	181
A.4	Impulse-Responses to an Investment-Specific Technology Shock.	182
A.5	Impulse-Responses to a Government Spending Shock.	182
A.6	Impulse-Responses to a Labor-Wedge Shock.	183
A.7	Impulse-Responses to a Risk Shock.	183
A.8	Impulse-Responses to an Equity Shock.	184
B.1	Model Predictions: US Great Recession	197

List of Tables

1.1	Calibrated Parameters	48
1.2	Estimated Parameters on U.S. Data - Model with Search Frictions	50
1.3	Second Moments: Data and Model	51
1.4	Variance Decomposition	55
1.5	Variance Decomposition	64
2.1	Descriptive Statistics	79
2.2	First Stage: Credit Cycle at the Output Peak and Financial Crises . . .	82
2.3	Financial Crises and Labor Market Recovery	84
2.4	Credit Recovery and Labor Market Recovery	86
2.5	Financial Crises and Labor Market Recovery—Additional Controls— De- veloped Market Economies	89
2.6	Financial Crises and Labor Market Recovery—Additional Controls – Emerging Market Economies	90
2.7	Financial Crises and Alternative Measures of Jobless Recovery, Developed Market Economies	91
2.8	Collateral Values and Jobless Recovery Developed Market Economies . .	98
3.1	Baseline Parameter Values	124
3.2	Means and Volatilities by Exchange Rate Regime	134
3.3	Welfare Costs by Exchange-Rate Regime	136
3.4	Second Moments – Data and Model	141

A.1	Sample of Recession Episodes	176
B.1	Sample Recession Episodes	194

Acknowledgements

I would like to thank Martín Uribe for his advice and support throughout this dissertation; ever since my arrival at Columbia, his belief in my work was a pillar on which this dissertation could stand. I would also like to thank the other members of my dissertation committee, Guillermo Calvo, Ricardo Reis, Stephanie Schmitt-Grohé, and Michael Woodford, for their guidance and advice. Each of them gave a different perspective to my dissertation and supported my work in an invaluable way.

My work also benefited from excellent comments and suggestions from Javier Bianchi, Saki Bigio, Patrick Bolton, Richard Clarida, Jennifer La'O, Emi Nakamura, Serena Ng, Jaromir Nosal, Bernard Salanié, Jon Steinson, Joseph Stiglitz, Ernesto Talvi, and all the faculty members of the Department of Economics and the Business School at Columbia University, as well as the visitors who provided very useful feedback on my work.

This dissertation was partly written at the Federal Reserve Bank of New York while I was a summer intern; I am grateful for their hospitality and for all the feedback from the members of the team.

I would also like to thank my classmates, Keshav Dogra, Ferran Elias, Mariana Garcia, Kyle Jurado, Sergei Kolbin, Wataru Miyamoto, Neil Mehrotra, Joan Monras, Thuy Lan Nguyen, Sebastian Rondeau, Samer Shousha, Dmitriy Sergeyev, and all the participants of the Economic Fluctuations Colloquium, Monetary Economics Colloquium, and Financial Frictions Colloquium at Columbia University for very valuable feedback and discussions. Special thanks to Diego Perez, a friend who supported my

work any time he could help.

Finally, I would like to thank my family and friends, who provided support and perspective to my work. I would like to especially thank Valentina Duque for joining me on this ride, and for bringing the best energy and enjoyment of all these years.

A la memoria de Jorge Ottonello

Chapter 1

Capital Unemployment, Financial Shocks, and Investment Slumps

Pablo Ottonello

1.1 Introduction

The U.S. Great Recession was followed by a persistent investment slump: Five years after the trough, investment rates remain below their historical average, and the stock of capital continues to fall with respect to its trend, constituting the most important contributor to persistently low economic activity (see [Hall, 2014](#), and [Figure 1.1](#)). The low levels of aggregate investment observed during the recovery from the U.S. Great Recession are challenging from the points of view of real and monetary models (see [Kydland and Zarazaga, 2012](#); [Del Negro, Giannoni and Patterson, 2012](#)). According to these large classes of models, the recovery should be characterized by high investment rates and rising stocks of physical capital.

The low-investment pattern exhibited by the recovery from the U.S. Great Recession is a salient characteristic of financial-crisis episodes across time and space. [Figure 1.1](#) shows evidence from a sample of 100 post-war recession episodes in advanced economies. Recoveries from financial crises are characterized by investment rates below the historical average and by a fall in capital stock with respect to its trend – as observed in the U.S. Great Recession.¹ This pattern is not, in fact, charac-

¹A “financial crisis” is defined as a recession episode in which a banking crisis event (as defined in

teristic of the average “regular” recession episode, in which investment rates recover with output and capital stock stabilizes close to its trend.

Motivated by this evidence, this paper constructs a general equilibrium framework in which financial shocks lead to investment slumps. The key idea in the model is that the production of new capital is affected by existing “capital unemployment” (i.e., owners of idle units of capital unable to find a firm willing to buy or rent these units to produce). After a negative financial shock (i.e., shocks to the net worth of the business sector or the risk of business projects), the share of unemployed capital is high; the economy, then, can achieve a better allocation by directing more resources to absorb existing unemployed capital into the production process and directing fewer resources to the accumulation of new capital, leading to low investment rates even after the shock has dissipated. The model’s main assumption, which leads to equilibrium capital unemployment, is that trade in physical capital occurs in a decentralized market characterized by search frictions, capturing costs that firms face when matching capital to business projects.

To assess the quantitative importance of the proposed mechanism, the paper constructs a stochastic business-cycle model with investment search frictions and capital unemployment. The model is estimated for the U.S. economy using Bayesian techniques and data prior to the U.S. Great Recession. It is shown that following a sequence of shocks such as those experienced by the U.S. economy in 2008 – and with no further shocks – the model predicts the persistence of aggregate investment and at least half of the output persistence observed in the aftermath of the U.S. Great Recession. Conducting the same exercise in a benchmark model without investment search frictions, the model predicts that both investment and output should be significantly

[Reinhart and Rogoff, 2009a](#)) takes place between the output peak and recovery point. Appendix [A.1](#) describes the sample construction and data used. The finding of investment lagging behind output recovery has been documented by [Calvo, Izquierdo and Talvi \(2006\)](#) in a sample of emerging-market sudden-stop crisis episodes. For other empirical studies characterizing financial-crisis episodes, see [Cerra and Saxena \(2008\)](#) and [Reinhart and Rogoff \(2009b, 2014\)](#).

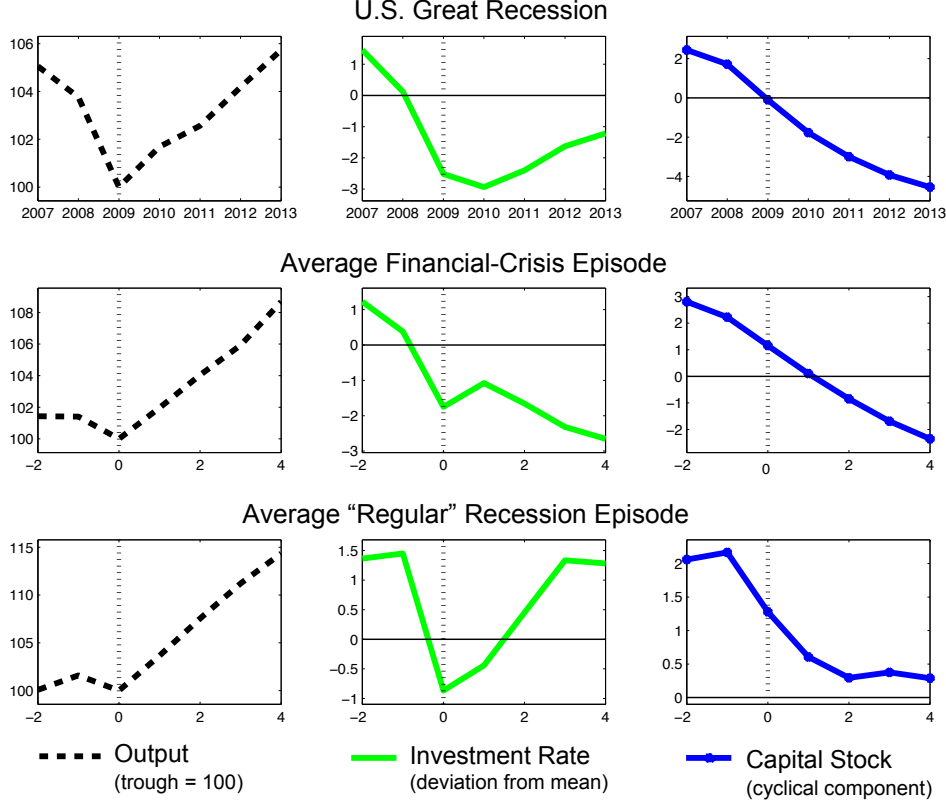


FIGURE 1.1: Financial Crises and Investment Slumps.

Note: Output refers to real, per capita, gross domestic product; investment rate refers to the ratio of gross fixed capital formation to gross domestic product; capital stock refers to the net stock of fixed assets. The sample of recession episodes includes 20 “financial crisis” episodes and 80 “regular” recession episodes for 22 advanced economies in the period 1950–2006. For each country included in the sample, investment rates are expressed in percent deviation from the mean over 1950–2013; the capital stock is expressed in percent deviation from a log-quadratic trend. In each recession episode, $t = 0$ denotes the output trough. The time unit for average episodes is a year. See Appendix A.1 for a description of the sample and data.

higher than the levels observed in the data, as noted in the previous literature.

The estimated model is also used to interpret the sources of U.S. business-cycle fluctuations. Results indicate that investment search frictions and capital unemployment are a relevant propagation mechanism for financial shocks: While these shocks account 33% of output fluctuations in the model with investment search frictions, they only account for 1% of output fluctuations in the benchmark real model without investment search frictions. Real models with financial frictions that distort firm purchases of capital can only assign a small role to financial shocks primarily because

observed fluctuations in aggregate investment do not imply large fluctuations in the stock of capital, which is the input to the production function (as discussed, for example in [Schwartzman, 2012](#); [Bigio, 2014](#)). In the framework developed in the present paper, the input to the production function is employed capital, which does fluctuate significantly in response to firm purchases of capital following financial shocks. The estimated model disciplines the fluctuations in capital unemployment with data on commercial real estate vacancy rates (office, retail, and industrial space). As shown in [Figure 1.2](#), the level and fluctuations in this measure of capital unemployment are comparable to those of U.S. labor unemployment.² The estimation attributes most of the fluctuations in capital unemployment to financial shocks, which have a large effect on firms' capital demand.

In the model search is directed, in the sense that sellers and buyers can search offers at a particular price, and the probability of finding a match depends on this price (see, for example, [Shimer, 1996](#); [Moen, 1997](#)). Search frictions in the physical capital market were first studied in a random search environment by [Kurmann and Petrosky-Nadeau \(2007\)](#). In a calibrated version of their model they show that these frictions are not a quantitatively relevant propagation mechanism of TFP shocks. The most important difference from their quantitative framework is the inclusion of financial shocks, that in the present paper account for most of the fluctuations in market tightness. In fact, if the present paper included only TFP shocks, it would also have concluded that search frictions in investment are not a relevant quantitative propagation mechanism once output fluctuation is matched, a result reminiscent to that found in [Shimer \(2005\)](#) for the labor market.

The directed-search framework for the physical-capital market developed in the present paper builds on those developed for the labor market in [Shi \(2009\)](#), [Menzio](#)

²Figure [A.1](#) of [Appendix A.2](#) shows that measures of capital unemployment available to Euro economies experiencing deep financial crises (Greece, Ireland, Portugal, and Spain) also show a large increase in capital unemployment, comparable to that of labor unemployment.

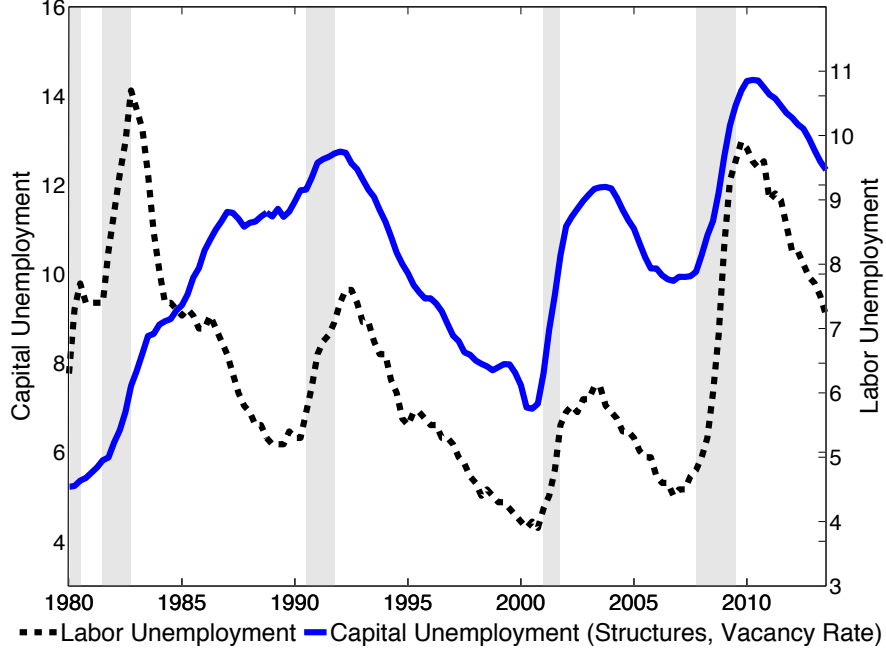


FIGURE 1.2: U.S. Unemployment of Physical Capital and Labor, 1980–2013.

Note: Capital unemployment (structures) constructed based on vacancy rates of office, retail and industrial units. Data source: CBRE and REIS. See Appendix A.1 for details. Labor unemployment refers to the civilian unemployment rate. Data source: Federal Reserve of Saint Louis. Data is expressed in percent. Shadow areas denote NBER (peak to trough) recession dates.

and Shi (2010, 2011), Schaal (2012) and Kaas and Kircher (2013). Studying these frictions for the physical capital market provides two novel mechanisms: First, it provides a new interaction between the production of capital and capital unemployment. The existence of high capital unemployment leads to a lower accumulation of new capital goods, while existing units are absorbed into production. This mechanism is not present in labor-market models in which population is generally assumed to be constant or exogenous. Second, because physical capital is not only a factor of production, but can also be used by firms as collateral for loans (see, for example, Kiyotaki and Moore, 1997; Geanakoplos, 2010), fluctuations in capital unemployment interact with financial shocks in a way not seen in the labor market.

The framework developed in this paper can also be used to study capital reallocation. This is done by extending the model to allow for heterogeneity in capital

match-specific productivity. This extension allows a characterization not only of the transition of capital from unemployment to employment, but of the transition of capital from employment to employment, since it adds a motive for trading unmatched capital while it remains employed (similar to “on the job search” in the labor-market literature). As shown in [Shi \(2009\)](#) and [Menzio and Shi \(2011\)](#) for the labor market, the directed-search structure of the model is especially suitable to studying employment–employment transitions resulting from heterogeneity in match-specific productivity. The paper shows that capital reallocation is procyclical in this framework, as in the data (see [Ramey and Shapiro, 1998](#); [Eisfeldt and Rampini, 2006](#)). This is because negative shocks are associated with fewer capital purchases, making it harder for sellers of employed capital to find buyers.

Layout. The remainder of the paper is organized as follows. Section [1.2](#) discusses the relationship with the literature. Section [1.3](#) introduces investment search frictions and capital unemployment into a simple neoclassical growth model, and presents the main mechanism relating capital unemployment to capital accumulation. Section [1.4](#) builds a quantitative business-cycle model including search frictions in investment. Section [1.5](#) presents the model estimation and the quantitative results. Section [1.6](#) studies capital reallocation in the framework of the model. Section [1.7](#) concludes and discusses possible extensions.

1.2 Relationship with the Literature

This section discusses the contribution of the present paper from the perspective of four strands of the literature.

Financial Shocks and Macroeconomic Fluctuations. This paper builds on the growing body of literature that studies the effect of financial shocks on macroe-

conomic fluctuations. The study of the implications of financial frictions has a long tradition in macroeconomics (for a recent survey, see [Brunnermeier, Eisenbach and Sannikov, 2012](#)). Following the Great Recession, a number of studies have shown that shocks that affect the severity of financial frictions can have a large impact on aggregate fluctuations (see, for example, [Mendoza, 2010](#); [Arellano, Bai and Kehoe, 2012](#); [Jermann and Quadrini, 2012](#); [Gertler and Kiyotaki, 2013](#); [Christiano, Motto and Rostagno, 2014](#)).

The present paper contributes to this literature with a new financial-shock propagation mechanism by introducing the possibility of capital unemployment, whose fluctuations are mostly driven by this type of shock. The propagation mechanism proposed for financial shocks in this paper provides two novel dimensions to this literature. First, the relevant role assigned to financial shocks does not rely on price or wage stickiness, and holds in the context of a real model that would assign a small role to financial shocks in the absence of investment search frictions. The role of financial shocks is a key discussion in the business-cycle literature and an important source of discrepancy between real and monetary models, with the latter attributing a much larger effect to these shocks than the former (as discussed in [Christiano, Motto and Rostagno, 2014](#)). The present paper shows that an important part of the discrepancy between these two branches of the literature can be reconciled by introducing investment search frictions. In a second contribution to this literature, the present paper provides a mechanism whereby financial shocks are followed by investment slumps, as documented in Figure 1.1.³

Investment Dynamics. By studying investment slumps following financial shocks, this paper relates to the large body of literature studying aggregate investment dy-

³[Queralto \(2013\)](#) constructs a quantitative framework in which financial crises have persistent effects on economic activity. Since that mechanism relies on endogenous TFP growth, the findings are complementary to those presented in the present paper.

namics (see, for example, [Caballero, 1999](#); [Cooper and Haltiwanger, 2006](#)). In particular, the mechanism of this paper is consistent with the empirical findings that attribute a key role to financial factors in aggregate investment (see for example [Gilchrist, Sim and Zakrajšek, 2014](#)).

Since the set of financial shocks studied in this paper include a shock to the idiosyncratic cross-sectional uncertainty of the quality of capital, the findings of this paper are also related to the recent branch of the literature studying the effect of uncertainty shocks on aggregate investment and economic activity (see, for example, [Bloom, Bond and Van Reenen, 2007](#); [Bloom, 2009](#); [Bloom et al., 2012](#)). In these papers, uncertainty leads firms to adopt a “wait-and-see” strategy, contracting investment until uncertainty is revealed. The difference with these papers is that the wait-and-see strategy only implies a short-lived pause in investment: investment recovers after uncertainty dissipates. The present paper studies a mechanism by which, if these shocks lead to a significant increase in capital unemployment, the effects in investment can be persistent, as observed in the U.S. Great Recession and the typical financial crisis episode.

In a recent independent work, [Rognlie, Shleifer and Simsek \(2014\)](#) also study persistent falls in investment such as the one following the U.S. Great Recession. The key ingredients to their model are an overbuilding of residential capital, nominal rigidities, and the zero lower bound.⁴ Therefore the mechanism of their paper and that of the present paper are complementary interpretations of the investment slump following the U.S. Great Recession. The result of the present paper also applies to financial crises in which monetary policy is not constrained by the zero lower bound and to those in which a residential overbuilding does not take place.

⁴See also [Schmitt-Grohé and Uribe \(2012b\)](#) and [Eggertsson and Mehrotra \(2014\)](#) for related papers studying the persistence of the Great Recession associated to the zero lower bound.

Search Frictions. By modeling capital unemployment in a search theoretical framework, this paper relates to the extensive literature studying search frictions in labor, assets, and goods markets. The relationship with the literature on search frictions in the labor market was discussed in Section 1.1. Given that physical capital is both a good and an asset, the search frictions studied in this paper are also related to those of goods markets or other asset markets. With regard to goods markets, [Bai, Rios-Rull and Storesletten \(2012\)](#) recently studied search frictions that affect the purchase of investment goods, as in the present paper. Unlike the present paper, these frictions only affect the flow of production and not the stock of existing capital units (which is the main feature of capital unemployment).

In other asset markets, a number of contributions have shown how search frictions affect the liquidity and returns of assets (for a recent survey, see [Lagos, Rocheteau, and Wright, 2014](#)). In the housing market, search frictions have been used to explain fluctuations in prices, trading and vacancy rates (see, for example, [Wheaton, 1990](#); [Krainer, 2001](#); [Caplin and Leahy, 2011](#); [Piazzesi, Schneider and Stroebel, 2013](#)). The main difference with respect to these contributions is that the physical capital considered in the present paper is a productive asset, and therefore fluctuations in its unemployment have a direct relationship with economic activity and firms' investment.

Capital Utilization. The effect of capital unemployment on economic activity is related to that studied in the literature on variable capital utilization (for surveys on capital utilization, see [Winston, 1974](#); [Betancourt and Clague, 2008](#)). However, capital unemployment and capital utilization are two different concepts, related to different economic mechanisms. To clarify the difference between the two concepts, it is useful to define a set of categories to classify capital stock, similar to those used to classify the status of the labor force, summarized in Figure 1.3 (see, for

			Owner of Capital Searching to Sell or Rent	
			No	Yes
Capital Held by	Agent without Production Technology	Idle	1.	2.
			Capital Outside of the “Capital Force”	Unemployed Capital
	Agent with Production Technology	Not Idle	3.	4.
			Used Less Intensely	6.
			Employed Capital Not Searching	Employed Capital with “On the Job Search”
			7.	8.
			Used More Intensely	

FIGURE 1.3: A Classification of Capital Stock Status.

Note: The capital stock status is classified defining eight categories similar to those in the labor market (see, for example, [Bureau of Labor Statistics, 2014](#)). Employed capital (Regions 5, 6, 7 and 8) includes units of capital that have been used for production within a period. This includes capital temporarily idle as part of regular business operations, such as shift changes. Unemployed capital (Regions 2 and 4) includes units of capital that have not been used for production within the period and their owners have actively searched to sell or rent the capital unit. Employed and unemployed capital constitute the “capital force.” Capital outside the capital force (Regions 1 and 3) includes idle units that have not been used for production within the period and whose owners are not seeking buyers or renters.

example, [Bureau of Labor Statistics, 2014](#)). An unemployed unit of capital is a unit of capital that has not been used for production within the period and whose owners have actively searched to sell or rent the capital unit. Therefore, while capital utilization describes the intensity with which capital is used by firms that own or rent capital (a consumption decision), capital unemployment describes whether owners of idle capital are unable to sell or rent it (an investment decision). The difference between capital unemployment and capital utilization then parallels that between labor unemployment and labor hoarding (see, for example, [Burnside, Eichenbaum and Rebelo, 1993](#); [Sbordone, 1996](#)).

Being two different concepts, capital utilization and capital unemployment can

have different empirical measures. For instance, standard empirical measures of capital utilization relate to firms’ use of their production capacity.⁵ Empirical measures of capital unemployment would instead relate the share of physical capital (owned by either firms or households) that is idle and available in the market for sale or rent, such as this paper’s data collected from the commercial real estate market (see Figure 1.2). As illustrated in Appendix A.2 (Figure A.2) for recent U.S. recession episodes, these empirical measures of capital unemployment and capital utilization can have significantly different behaviors.

Capital utilization and capital unemployment can also be modeled differently. Models of capital utilization typically treat it as a control variable whose choice, related to utilization costs, can be described as an intensive margin (e.g., a higher utilization rate causes greater depreciation, as in Calvo, 1975; Greenwood, Hercowitz and Huffman, 1988, Regions 5 and 7 of Figure 1.3) or as an extensive margin (e.g., less-productive units are left idle, as in Cooley, Hansen and Prescott, 1995; Gilchrist and Williams, 2000, Region 3 of Figure 1.3). Recent contributions using search frictions in the product market show that this variable can also be related to the probability of a firm finding customers (see, for example, Petrosky-Nadeau and Wasmer, 2011; Bai, Rios-Rull and Storesletten, 2012; Michaillat and Saez, 2013). In the present paper’s model, capital unemployment is a state variable. The key margins affecting the flows of unemployed capital to employment are the price of capital posted by sellers and the mass of capital buyers are willing to purchase at a given price (transition from Regions 2 and 8 to Regions 5 and 7 of Figure 1.3).

For this reason, this paper will show that different factors affect fluctuations in

⁵For the U.S., the Federal Reserve Board estimates capacity utilization for industries in manufacturing (see Federal Reserve Board, 2014, for a description of the methodology). Gorodnichenko and Shapiro (2011) use the Survey of Plant Capacity from the U.S. Census Bureau (2014) to construct data on capital utilization. Basu, Fernald and Kimball (2006), Basu et al. (2013), and Fernald (2012) provide estimates of factor utilization for the U.S. economy, capturing labor effort and the work week of capital.

capital utilization and capital unemployment and that different implications follow from explicitly modeling capital unemployment (such as the a low rate of investment when capital unemployment is high). Nevertheless, the concepts of capital utilization and capital unemployment can be seen as complementary. In fact, once the model with capital unemployment is extended to study capital reallocation (transitions from Regions 6 and 8 to Regions 5 and 7 of Figure 1.3), changes in the probability of selling capital units will affect firms' capital utilization rates.

1.3 Investment Search Frictions: Basic Framework

This section introduces investment search frictions into a simple neoclassical growth model. The framework abstracts from uncertainty, endogenous labor supply and other frictions – which will be later introduced in the quantitative model – to make the mechanism clear. Policy functions and transitional dynamics are studied, showing how capital accumulation is affected by existing capital unemployment. In the standard neoclassical growth model, the process of convergence from an initial capital stock below the steady state is characterized by a monotonic increase in the capital stock. By contrast, in the model with investment search frictions, if the initial total capital stock is below the steady state and the rate of capital unemployment is sufficiently high, the transitional dynamics for the capital stock are not monotonic, featuring an initial decrease and a subsequent increase. Therefore, if a shock leads to a sufficiently high level of capital unemployment, recovery is characterized by an investment slump, as documented in Figure 1.1.

1.3.1 Environment

Time is discrete and infinite, with four-stage periods. There is no aggregate uncertainty.

Goods. There are consumption and capital goods: Consumption goods are perishable; capital goods depreciate at a constant rate, $\delta > 0$. Capital can be traded in either of two states: matched or unmatched. Only matched capital can be used as input in the production of consumption goods.

Agents. The economy is populated by a unit mass of identical households and a unit mass of entrepreneurs. Households consume, produce unmatched physical capital and (inelastically) supply labor. The representative household has a continuum of infinitely lived members, a positive fraction of whom are entrepreneurs. Within each household there is perfect consumption insurance.⁶ Entrepreneurs have access to a technology to produce consumption goods, using matched capital and labor as inputs, and to a search technology to transform unmatched capital into matched capital. Capital produced by households begins unmatched. Only entrepreneurs can store matched capital. Capital held by entrepreneurs is denoted *employed capital*, and capital held by households is denoted *unemployed capital*.

Each period, entrepreneurs have a probability $\psi > 0$ of retiring from entrepreneurial activity. The fraction ψ of entrepreneurs who retire from entrepreneurial activity is replaced by a new identical mass of entrepreneurs from the households' members, so the population of entrepreneurs is constant. Retiring entrepreneurs' capital becomes unmatched and is transferred to the household. Divi-

⁶The assumption of large families follows [Merz \(1995\)](#), [Andolfatto \(1996\)](#) and, more recently, [Gertler and Karadi \(2011\)](#) and [Christiano, Motto and Rostagno \(2014\)](#). This assumption facilitates the work in Section 1.4, when financial frictions are introduced explicitly and entrepreneurs are endowed with net worth. In the current section, this assumption plays no role and is not different from a framework in which a representative firm produces consumption goods.

dends from entrepreneurial activity, resulting from capital purchases and production, are transferred each period to the household.

Physical capital markets. Trade of unmatched capital between entrepreneurs and households occurs in a decentralized market with search frictions. Search is directed, following a structure similar to those in [Menzio and Shi \(2010, 2011\)](#) for the labor market and in [Menzio, Shi and Sun \(2013\)](#) for the money market. In particular, this market is organized in a continuum of submarkets indexed by the price of unmatched capital, denoted x . Sellers (households) and buyers (entrepreneurs) can choose which submarket to visit. In each submarket, the market tightness, denoted $\theta(x)$, is defined as the ratio between the mass of capital searched by entrepreneurs and the mass of unemployed capital offered in that submarket. Households face no search cost. Visiting submarket x in period t , they face a probability $p(\theta_t(x))$ of finding a match, where $p : \mathbb{R}_+ \rightarrow [0, 1]$ is a twice continuously differentiable, strictly increasing, strictly concave function that satisfies $p(0) = 0$ and $\lim_{\theta \rightarrow \infty} p(\theta) = 1$. Entrepreneurs face a cost per unit searched denominated in consumption goods and denoted $c_s > 0$. Visiting submarket x in period t , they face a probability $q(\theta_t(x))$ of finding a match, where $q : \mathbb{R}_+ \rightarrow [0, 1]$ is a twice continuously differentiable, strictly decreasing function that satisfies $q(\theta) = \frac{p(\theta)}{\theta}$, $q(0) = 1$ and $\lim_{\theta \rightarrow \infty} q(\theta) = 0$. The cost of a unit of capital for entrepreneurs in submarket x is denoted Q^x (which includes two components: the price paid to the seller, x , and the search cost in submarket x).

Timing. Each period is divided into four stages: production, separation, search, and investment. In the *production stage*, entrepreneurs produce consumption goods using matched capital from the previous period; employed and unemployed capital depreciates. In the *separation stage*, a fraction ψ of entrepreneurs retires and their capital becomes unmatched. An identical mass of entrepreneurs begins entrepreneurial activity with no initial capital. In the *search stage*, entrepreneurs who

do not retire and new entrepreneurs purchase unmatched capital from households, and net dividends in terms of consumption goods are transferred. In the *investment stage*, households produce physical capital and consume, and retired entrepreneurs transfer their capital to households.

1.3.2 Households

Household preferences are described by the lifetime utility function

$$\sum_{t=0}^{\infty} \beta^t U(C_{i,t}), \quad (1.1)$$

where $C_{i,t}$ denotes consumption of household i in period t , $\beta \in (0, 1)$ is the subjective discount factor, and $U : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a twice continuously differentiable, strictly increasing, strictly concave function.

Unemployed capital held by household i evolves according to the law of motion

$$K_{i,t+1}^u = \int_0^{(1-\delta)K_{i,t}^u} (1 - p(\theta_t(x_{k,i,t}))) dk + \psi(1 - \delta)K_t^e + I_{i,t}, \quad (1.2)$$

where $K_{i,t}^u$ denotes the stock of unemployed capital held by household i at the beginning of period t , K_t^e denotes the stock of employed capital at the beginning of period t , $I_{i,t}$ denotes the household's investment in period t , and $x_{k,i,t}$ denotes the submarket in which unemployed capital unit k is listed by household i in period t . The first term of the right-hand side of equation (1.2) represents the depreciated mass of capital which was unemployed at the beginning of period t and was not sold to entrepreneurs for a given market tightness, $\theta_t(x)$, and submarket choice $x_{k,i,t}$. The second term of the right-hand side of equation (1.2) represents the mass of employed capital transferred from retired entrepreneurs to households. The third term represents the addition (subtraction) to unemployed capital stock from investment.

The sequential budget constraint of household i is given by

$$C_{i,t} + I_{i,t} = \int_0^{(1-\delta)K_{i,t}^u} p(\theta_t(x_{k,i,t}))x_{k,i,t} dk + W_t \bar{h} + \tilde{\Pi}_{i,t}, \quad (1.3)$$

where W_t denotes the wage rate in period t , \bar{h} denotes the household (inelastic) supply of hours of work to the labor market, and $\tilde{\Pi}_{i,t}$ denotes net transfers in terms of consumption goods from entrepreneurs to household i in period t – described further in the next section. The left-hand side of equation (1.3) represents the uses of income: consumption and investment. The right-hand side of the equation represents the sources of income: selling unmatched capital in the decentralized market, labor income, and transfers from entrepreneurs.

Household i 's problem is then to choose plans for $C_{i,t}$, $I_{i,t}$, $K_{i,t+1}^u$, and $x_{k,i,t}$ that maximize utility (1.1), subject to the sequence of budget constraints (1.3), the accumulation constraints for unemployed capital (1.2), given the initial levels of capital, $K_{i,0}^u$ and K_0^e , the given sequence of net transfers, $\tilde{\Pi}_{i,t}$, and the given sequence of market-tightness functions, $\theta_t(x)$. Denoting $\Lambda_{i,t}$ the Lagrange multiplier associated with the budget constraint (1.3), in an interior solution, the optimality conditions are (1.2) and (1.3), and the first-order are conditions

$$\Lambda_{i,t} = U'(C_{i,t}), \quad (1.4)$$

$$\Lambda_{i,t} = \beta \Lambda_{i,t+1} (1 - \delta) [p(\theta_{t+1}(x_{i,t+1}^u)) x_{i,t+1}^u + (1 - p(\theta_{t+1}(x_{i,t+1}^u)))] , \quad (1.5)$$

$$-p(\theta(x_{i,t}^u)) = p'(\theta(x_{i,t}^u)) \theta'_t(x_{i,t}^u) (x_{i,t}^u - 1), \quad (1.6)$$

where $x_{i,t}^u$ denotes household i 's choice of submarket for unmatched capital in period t , the unit of capital subindex, k , has been dropped because the optimality condition with respect to the choice of submarket, $x_{i,t}$, is the same for all units of capital.

1.3.3 Entrepreneurs

Entrepreneurs have access to a technology to produce consumption goods that uses matched capital as input:

$$Y_{j,t} = A_t F(K_{j,t}^e, h_{j,t}) = A_t (K_{j,t}^e)^\alpha (h_{j,t})^{1-\alpha}, \quad (1.7)$$

where $Y_{j,t}$ denotes output produced by entrepreneur j in period t , $K_{j,t}^e \geq 0$ denotes the stock of matched capital held by entrepreneur j at the beginning of period t , $h_{j,t}$ denotes hours of work employed by entrepreneur j in period t , A_t is an aggregate productivity factor affecting the production technology in period t .

The entrepreneur's objective is to maximize the present discounted value of dividends distributed to households:

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \Pi_{j,t+s}^e, \quad (1.8)$$

where $\Pi_{j,t}^e$ denotes net dividends paid by entrepreneur j to the household in period t , E_t denotes the expectation conditional on the information set available at time t (the expected value is over the idiosyncratic retirement shock), and the household's subindex, i , in the shadow value Λ_t has been dropped since the first-order conditions of the household's problem are the same for all households. Net dividends of entrepreneur j are defined by the flow-of-funds constraint:

$$\Pi_{j,t}^e = A_t F(K_{j,t}^e, h_{j,t}) - W_t h_{j,t} - (1 - \psi_{j,t}) \left[\int_x Q_t^x \ell_{j,t}^{e,x} dx \right] + \psi_{j,t} (1 - \delta) K_{j,t}^e, \quad (1.9)$$

where $\ell_{j,t}^{e,x} \geq 0$ denotes the mass of capital purchased by entrepreneur j in sub-market x in period t , and the stochastic variable $\psi_{j,t} \in \{0, 1\}$ takes the value 1 if entrepreneur j retires from entrepreneurial activity in period t and 0 otherwise, and satisfies $E_{t-1}(\psi_{j,t}) = \psi \forall t, j$. The three terms in the right-hand side of equation (1.9) represent the sources of net dividends transferred from entrepreneurs to households: The first term represents the output in terms of consumption goods produced by entrepreneur j in period t . The second term denotes the net purchase of physical capital, expressed in consumption units, that entrepreneur j makes in the case of not retiring in period t . The last term represents the transfer of unmatched capital that entrepreneur j makes to households in the case of retiring in period t . The first two terms define the net transfer, in terms of consumption goods, that entrepreneur j

makes to households in period t : $\tilde{\Pi}_{j,t}^e \equiv A_t F(K_{j,t}^e) - W_t h_{j,t} - (1 - \psi_{j,t}) \left[\int_x Q_t^x \iota_{j,t}^{e,x} dx \right]$ (see the household's budget constraint (1.3)).

By the law of large numbers, the cost per unit of capital, of mass $\iota_{j,t}^{e,x}$, purchased in the submarket x of the decentralized market is given by

$$Q_t^x = x + \frac{c_s}{q(\theta_t(x))}. \quad (1.10)$$

The right-hand side of equation (1.10) represents the two components of the cost of a unit of capital in the decentralized market: the price paid to the seller, x , and the search cost, $\frac{c_s}{q(\theta_t(x))}$.

The stock of matched capital for entrepreneur j , who has the opportunity to invest in period t , evolves according to the law of motion

$$K_{j,t+1}^e = (1 - \delta)K_{j,t}^e + \int_x \iota_{j,t}^{e,x} dx. \quad (1.11)$$

Denote the period in which entrepreneur j enters entrepreneurial activity as t_{0j} , and assume entrepreneurs enter entrepreneurial activity with no initial matched capital; that is, $K_{j,t_{0j}}^e = 0 \forall t_{0j} \geq 0$.⁷

Entrepreneur j 's problem, is then to choose plans for $K_{j,t+1}^e$, $\iota_{j,t}^{e,x}$, and $h_{j,t}$ that maximize the present discounted value of dividends (1.8) subject to the sequence of flow-of-funds constraints (1.9), the accumulation constraints for matched capital (1.11), and the nonnegativity constraints for capital purchases in the decentralized market ($\iota_{j,t}^{e,x} \geq 0$), given the initial level of matched capital, $K_{j,t_{0j}}^e$, the given sequence of aggregate productivity A_t , the given sequence of prices, W_t , and the given sequence of market-tightness functions, $\theta_t(x)$. Denoting the Lagrange multiplier associated with the budget constraint (1.11) in period $t+s$ with $Q_{j,t+s} \frac{\Lambda_{t+s}}{\Lambda_t}$ and by $\Xi_{j,t+s}^x \frac{\Lambda_{t+s}}{\Lambda_t}$ the Lagrange multiplier associated with the nonnegativity constraint for capital purchases in submarket x in period $t+s$, the optimality conditions are (1.9), (1.11), $\iota_t^{e,x} \geq 0$,

⁷A mass one of entrepreneurs starts period 0 with a stock of matched capital K_0^e .

the first-order conditions – which, after some operations, can be expressed as

$$h_{j,t} = \left(\frac{(1-\alpha)A_t}{W_t} \right)^{\frac{1}{\alpha}} K_{j,t+1}^e \quad (1.12)$$

$$\Lambda_t Q_{j,t} = \beta \Lambda_{t+1} \left[r_t^k + (1-\delta)(\psi + (1-\psi)Q_{j,t+1}) \right], \quad (1.13)$$

$$Q_t^x = Q_{j,t} + \Xi_{j,t}^x, \quad (1.14)$$

and the complementary slackness conditions,

$$\Xi_{j,t}^x \geq 0, \quad \iota_{j,t}^{e,x} \Xi_{j,t}^x = 0, \quad (1.15)$$

for all x , where the net revenues from production per unit of matched capital are defined by $r_t^k \equiv \alpha \left(\frac{1-\alpha}{W_t} \right)^{\frac{1-\alpha}{\alpha}} A_t^{\frac{1}{\alpha}}$.

1.3.4 Equilibrium

The entrepreneurs' optimality conditions, (1.14) and (1.15), imply that, in equilibrium, any submarket visited by a positive number of entrepreneurs must have the same cost per unit of capital, and entrepreneurs will be indifferent among them. Formally, for all x ,

$$\theta_t(x) \left(x + \frac{c_s}{q(\theta_t(x))} - Q_t \right) = 0. \quad (1.16)$$

where the entrepreneur's subindex, j , has been dropped in the shadow value Q_t because the optimality conditions (1.13)–(1.15) are the same for all entrepreneurs. This condition determines the equilibrium market-tightness function: For all $x < Q_t$,

$$\theta_t(x) = q^{-1} \left(\frac{c_s}{Q_t - x} \right). \quad (1.17)$$

For all $x \geq Q_t$, $\theta_t(x) = 0$: capital units listed above the value of capital for entrepreneurs remain unmatched.

Using the definition of market tightness, the law of large numbers, and the fact that a household's choice of submarket, $x_{k,i,t}$ is the same for all units of capital,

k , and all households, i , the flow of capital that transitions from unemployment to employment is given by $p(\theta_t(x_t^u))(1 - \delta) \int_0^1 K_{i,t}^u di = \int_0^1 \iota_{j,t}^{e,x^u} dj = \int_0^1 \int_x \iota_{j,t}^{e,x} dx dj$. Aggregating the entrepreneurs' capital-accumulation constraints provides a law of motion for employed capital:

$$K_{t+1}^e = (1 - \psi)(1 - \delta)K_t^e + p(\theta_t(x_t^u))(1 - \delta)K_t^u. \quad (1.18)$$

where the aggregate stock of employed and unemployed capital in period t are defined, respectively, by $K_t^e \equiv \int_0^1 K_{j,t}^e dj$ and $K_t^u \equiv \int_0^1 K_{i,t}^u di$.

From the household's capital-accumulation constraint (1.2), and using again the law of large numbers and the fact that the choice of submarket, $x_{k,i,t}$, is the same for all units of capital, k , and all households, i , a law of motion for unemployed capital is obtained:

$$K_{t+1}^u = (1 - p(\theta_t(x_t^u)))(1 - \delta)K_t^u + \psi(1 - \delta)K_t^e + I_t, \quad (1.19)$$

where aggregate investment is defined by $I_t \equiv \int_0^1 I_{i,t} di$.

The capital-unemployment rate at the beginning of period t can then be defined as

$$k_t^u \equiv \frac{K_t^u}{K_t}, \quad (1.20)$$

where $K_t \equiv K_t^e + K_t^u$ denotes total aggregate capital stock at the beginning of period t .

Labor-market clearing requires $\int_j h_{j,t} dj = \bar{h}$. Aggregating the households' budget constraints and the entrepreneurs' flow-of-funds constraints and using the entrepreneurs' optimality conditions and the laws of motion for employed and unemployed capital provides the economy's resource constraint:

$$C_t + I_t + c_s \theta_t(x_t^u)(1 - \delta)K_t^u = A_t F(K_t^e, \bar{h}). \quad (1.21)$$

where aggregate consumption is defined by $C_t \equiv \int_0^1 C_{i,t} di$.

The competitive equilibrium in this economy can then be defined as follows.

Definition 1 (Competitive equilibrium). *Given initial conditions for employed and unemployed capital, K_0^e and K_0^u , and sequences of aggregate productivity, A_t , a competitive equilibrium is a sequence of individual allocations and shadow values $\{(C_{i,t}, I_{i,t}, K_{i,t+1}^u, x_{i,t}^u)_{i \in [0,1]}, (K_{j,t+1}^e, \ell_{j,t}^{e,x}, h_{j,t})_{j \in [0,1]}\}$, $\{(\Lambda_{i,t})_{i \in [0,1]}, (Q_{j,t})_{j \in [0,1]}\}$, aggregate allocations $\{C_t, I_t, K_{t+1}^e, K_{t+1}^u\}$, prices $\{W_t\}$, and market-tightness functions $\{\theta_t(x)\}$ such that*

- (i) *The individual allocations and shadow values solve the household's and entrepreneur's problems at the equilibrium prices and equilibrium market-tightness functions for all i and j .*
- (ii) *The market-tightness function satisfies (1.16) for all x .*
- (iii) *The labor market clears.*

1.3.5 Characterizing Equilibrium

Efficiency. Given the directed-search structure of the decentralized market, it can be shown that the competitive equilibrium is efficient in the sense that its allocation coincides with the solution a social planner would select when facing the same technological constraints as those faced by private agents, including search effort. Efficiency is defined and established in the following definition and proposition.

Definition 2 (Efficient allocation). *A sequence of allocations, $\{C_t, I_t, K_{t+1}, k_{t+1}^u, \theta_t^u\}$, is efficient if it solves the following social planner's problem.*

$$\max_{\{C_t, I_t, K_{t+1}, k_{t+1}^u, \theta_t^u\}} \sum_{t=0}^{\infty} \beta^t U(C_t), \quad (1.22)$$

$$s.t. \quad C_t + I_t + c_s \theta_t^u (1 - \delta) k_t^u K_t = A_t F((1 - k_t^u) K_t, \bar{h}), \quad (1.23)$$

$$K_{t+1} = (1 - \delta) K_t + I_t, \quad (1.24)$$

$$(1 - k_{t+1}^u) K_{t+1} = [(1 - \psi)(1 - k_t^u) + p(\theta_t^u) k_t^u] (1 - \delta) K_t, \quad (1.25)$$

given initial conditions for capital stock and capital-unemployment rate, K_0 and k_0^u , and sequences of aggregate productivity, A_t .

Proposition 1. *The competitive equilibrium is efficient.*

Proof. See Appendix A.4. ■

Denoting the Lagrange multiplier of resource constraint (1.23) as Λ_t^{sp} , and the Lagrange multiplier for employed-capital law of motion (1.25) as $(Q_t^{\text{sp}} - 1)\Lambda_t^{\text{sp}}$, the optimality conditions of the social planner's problem are (1.23)–(1.25), and the first-order conditions, that after operating, can be expressed as

$$U'(C_t) = \Lambda_t^{\text{sp}}, \quad (1.26)$$

$$c_s = p'(\theta_t^u)(Q_t^{\text{sp}} - 1), \quad (1.27)$$

$$\Lambda_t^{\text{sp}} Q_t^{\text{sp}} = \beta \Lambda_{t+1}^c \{A_{t+1} F_1(K_{t+1}^e, \bar{h}) + (1 - \delta)[\psi + Q_t^{\text{sp}}(1 - \psi)]\}, \quad (1.28)$$

$$\Lambda_t^{\text{sp}} = \beta \Lambda_{t+1}^{\text{sp}} (1 - \delta) \{(1 - p(\theta_{t+1})) + Q_{t+1}^{\text{sp}} p(\theta_{t+1}) - c_s \theta_{t+1}\}. \quad (1.29)$$

Equation (1.26) states that the social planner equates the marginal utility of consumption with the social shadow value of wealth, Λ_t^{sp} . Equation (1.27) states that the planner equates the social marginal costs and benefits of increasing market tightness: The left-hand side of equation (1.27) represents the social marginal cost of increasing the market tightness, which is given by the cost c_s per unit searched; the right-hand side of equation (1.27) represents the social marginal benefit of increasing market tightness, which is the product of two terms: the marginal increase in the probability of matching unemployed capital, given by $p'(\theta_t^u)$, and the shadow value of employed capital in consumption units, $(Q_t^{\text{sp}} - 1)$. Equation (1.28) states that the planner equates the social marginal cost of increasing the employment rate of capital in period t , given by $\Lambda_t^{\text{sp}} Q_t^{\text{sp}}$, with the expected discounted social marginal benefit of increasing the capital employment rate in period $t + 1$, given by the right-hand side of equation (1.28), which includes the marginal product of employed capital (given by

$A_{t+1}F_1(K_{t+1}^e, \bar{h})$ and the expected depreciated value of a unit of employment capital (given by $(1 - \delta)[\psi + Q_t^{\text{sp}}(1 - \psi)]$). Finally, equation (1.29) states that the social planner equates the social marginal cost of increasing the capital stock in period t , given by Λ_t^{sp} , with the expected discounted social marginal benefit of increasing the capital stock in period $t + 1$ given by the right-hand side of equation (1.29). Since newly produced capital is unemployed, the marginal benefit is that of a consumption unit with probability $[1 - p(\theta_{t+1}^u)]$, and that of an employed unit of capital (given by $\Lambda_{t+1}^{\text{sp}}Q_t^{\text{sp}}$) with probability $p(\theta_{t+1}^u)$, net of search costs (given by $c_s\theta_{t+1}^u$).

Policy functions and transitional dynamics. This section studies the policy functions of the social planner's problem (1.22), and the resulting process of convergence from an initial capital stock and capital-unemployment rate to the steady-state path, assuming that aggregate technology is constant over time.

Figure 1.4 shows decision rules for next-period capital stock and next-period capital-unemployment rate, as a function of the two state variables: current capital stock and current capital-unemployment rate.⁸ In each panel, one state variable varies on the horizontal axis and the others are fixed at a given specified value. If the current share of unemployed capital is at its steady-state level, the planner's decision rules for next-period capital are similar to those of the standard neoclassical growth model: Increasing the capital stock for levels of current capital stock below the steady state, decreasing the capital stock for current values of capital above the steady-state level, as depicted in the top-left panel of Figure 1.4.

This pattern no longer holds if the capital-unemployment rate is above its steady-state level. As shown on the top-left panel of Figure 1.4, for a sufficiently high level of the current capital-unemployment rate, next-period's optimal capital stock is below its current level even for levels below the steady state. The reason for this

⁸Functional forms used were those of Section 1.5. Parameter values were set to those used as priors in the quantitative analysis of Section 1.5.

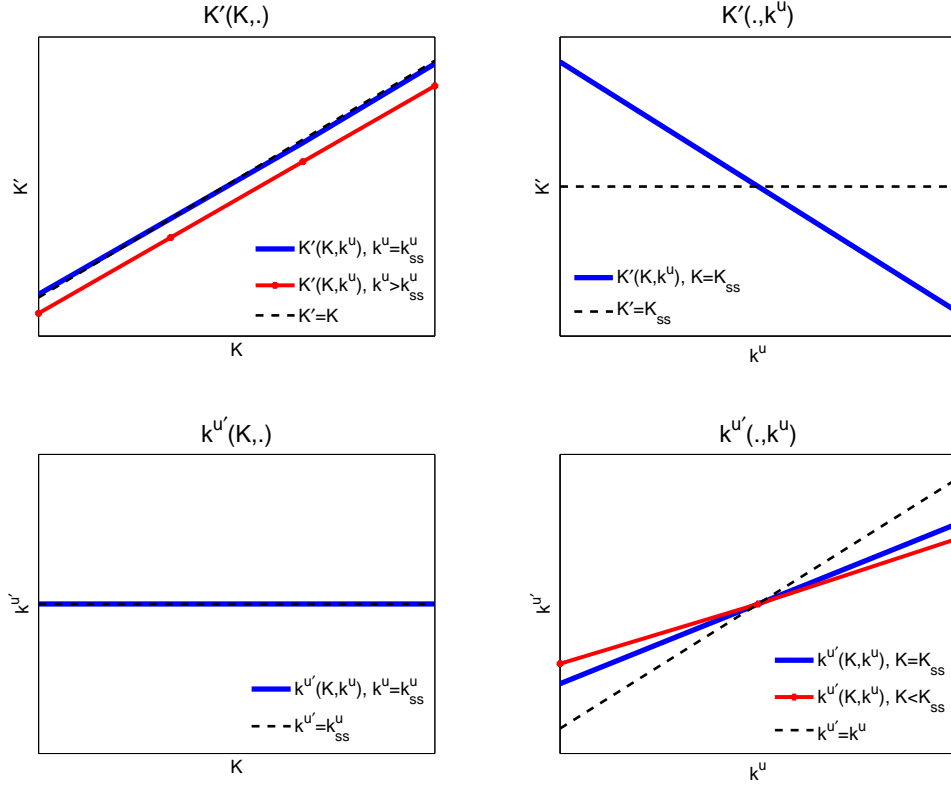


FIGURE 1.4: Policy Functions.

Note: Decision rules in the social planner's problem (1.22). In each panel, only one state variable varies on the horizontal axis. The other state variables is fixed at a given specified value.

is that, as depicted on the top-right panel of Figure 1.4, next-period capital stock is a decreasing function of the current share of unemployed capital. For instance, if the stock of capital is at its steady-state level, but the share of unemployed capital is above its steady-state level, the social planner chooses to decrease the capital stock. This is because, if the share of unemployed capital is above its steady state, the social planner wants to reduce next-period share of unemployed capital (see bottom-right panel of Figure 1.4). In the framework of the present paper, the production of new capital goods only increases the stock of unemployed capital (see equation (1.2)). For a given level of consumption, by reducing the stock of capital, the social planner can dedicate more resources to matching, and reduce the share of unemployed capital.

As implied by the policy functions, the transitional dynamics to the steady state, starting from a stock of capital below the steady state depends on the initial share

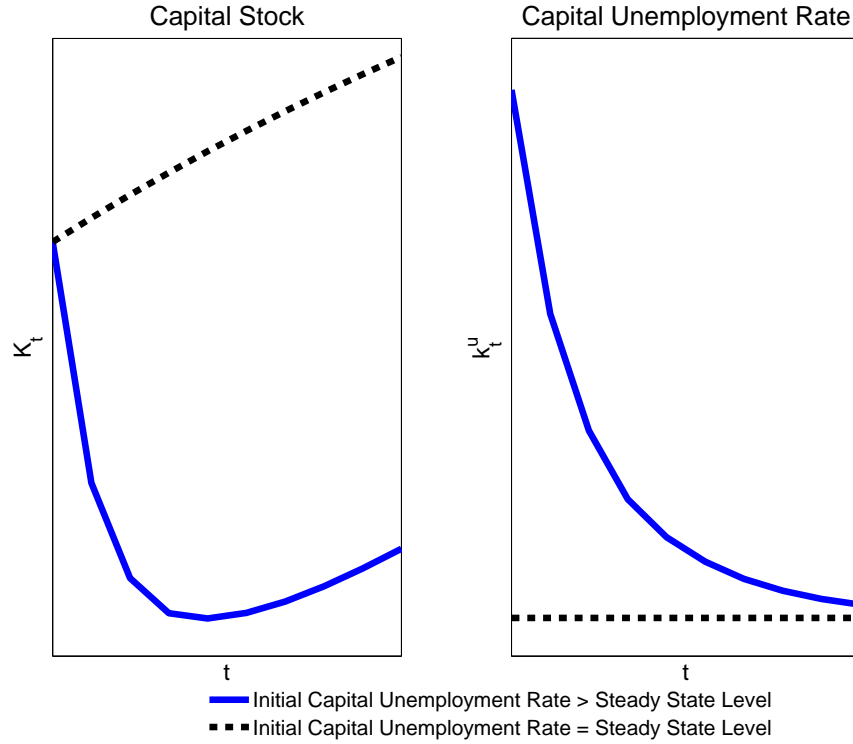


FIGURE 1.5: Transitional Dynamics and Initial Capital-Unemployment Rate.
Note: Transitional dynamics from initial capital stock (K_0) below the steady-state level for two alternative values of the initial capital-unemployment rate level (k_0^u).

of capital unemployment. As shown in Figure 1.5, starting from an initial share of unemployed capital equal to the steady-state level, the stock of capital increases monotonically, as it would in the standard neoclassical growth model. However, when the initial share of capital unemployment is sufficiently high, the stock of capital first decreases, and then increases to catch up with the steady-state level. Capital unemployment provides a reason why the recovery from a negative shock can be characterized by an investment slump (as shown in Figure 1.1). The remaining task is then to study which shocks can lead to a significant increase in the capital-unemployment rate. This will be analyzed quantitatively in Section 1.5.

To further study the economic mechanism induced by the investment search friction, Appendix A.3 considers a prototype economy with time-varying wedges (in the spirit of Chari, Kehoe and McGrattan, 2007), and maps the equilibrium of the

economy with search frictions in investment to wedges in the prototype economy.

1.4 A Quantitative Business-Cycle Model with Investment Search Frictions

This section extends the basic framework of Section 1.3 to a stochastic business-cycle environment to quantitatively study the proposed mechanism. The model includes financial frictions and two shocks related to the severity of the financial frictions that have been studied in the literature as having an important role in U.S. business cycles and in the Great Recession: shocks to the cross-sectional idiosyncratic uncertainty and to the business sector's net worth (Christiano, Motto and Rostagno, 2014; Jermann and Quadrini, 2012). It also features other frictions and shocks that the literature has shown to be relevant sources of business-cycle fluctuation in the U.S. economy (see Smets and Wouters, 2007; Justiniano, Primiceri and Tambalotti, 2010, 2011; Schmitt-Grohé and Uribe, 2012a). In particular, the model incorporates investment-adjustment costs, variable capital utilization, internal habit formation in consumption, and four other structural shocks: neutral productivity, investment-specific productivity, government spending and preferences.

1.4.1 Environment

Goods. As in Section 1.3, consumption goods are perishable, and capital goods depreciate at a rate $\delta > 0$. Capital goods can be traded in either of two states: matched or unmatched. Only matched capital can be used as input in the production of consumption goods.

Agents. The economy is populated by a unit mass of identical households, a unit mass of entrepreneurs, and an arbitrary large number of financial intermediaries (see

Figure 1.6). Households consume, supply labor, produce unmatched physical capital and purchase bonds issued by financial intermediaries. As in Section 1.3, the representative household has a continuum of infinitely lived members, with a positive fraction of them being entrepreneurs. Within each household, there is perfect consumption insurance. Entrepreneurs have access to a technology to produce consumption goods, using matched capital and labor as inputs, and to a search technology to transform unmatched capital into matched capital. Capital produced by households begins unmatched. Only entrepreneurs can store matched capital.

Unlike in Section 1.3, entrepreneurs cannot finance their purchases of capital with direct transfers from households. Instead, entrepreneurs purchase capital each period by borrowing from financial intermediaries and by using their own net worth.

Each period, an entrepreneur has a probability $\bar{\psi} > 0$ of retiring from entrepreneurial activity. The fraction $\bar{\psi}$ of entrepreneurs that retires from entrepreneurial activity each period is replaced by a new equal mass of entrepreneurs from the households' members. New entrepreneurs start entrepreneurial activity with an exogenous and stochastic stock of net worth transferred from the households. Retiring entrepreneurs' capital becomes unmatched and is traded with households, and their net worth, after selling the unmatched capital, is transferred to their households.

An unrestricted mass of financial intermediaries can enter the economy each period. They can sell bonds to households and lend to entrepreneurs for capital-good purchases. Additionally, the economy includes a government that conducts fiscal policy.

Markets. The economy has four competitive markets: goods, labor, physical capital and credit (see Figure 1.6). The goods and labor markets are frictionless. The market for physical capital is characterized by search frictions. The credit market is characterized by frictions associated with asymmetric information in lending. Further

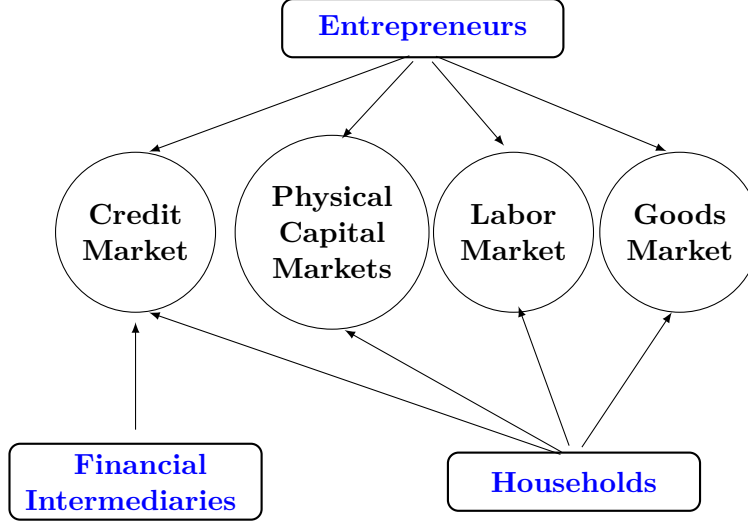


FIGURE 1.6: Agents and Markets.

details on the frictions that characterize the credit and physical-capital markets are provided below.

Credit market. Lending to entrepreneurs is assumed to entail an agency problem associated with asymmetric information and costly state verification ([Townsend, 1979](#)). In particular, entrepreneurs face an idiosyncratic shock whose realization is private information and can only be known by the lender through costly verification.

Following [Bernanke, Gertler and Gilchrist \(1999\)](#), it is assumed that the idiosyncratic shock is an i.i.d. shock to the quality of capital, denoted ω , whose realization is known by neither entrepreneurs nor financial intermediaries when lending occurs. Entrepreneurs finance the purchase of capital partly by borrowing and partly from their own net worth. The set of contracts offered to entrepreneurs, (Z_{t+1}, D_{t+1}) , specifies an aggregate state-contingent interest rate, Z_{t+1} , for each loan amount, D_{t+1} , to be repaid in case of no default. In the case of default, the financial intermediary seizes the entrepreneur's assets. It is further assumed that the capital held by the entrepreneur becomes unmatched in the event of default. This form of contract implies in each period that a cutoff value exists for the realization of ω , denoted $\bar{\omega}_t$, below

which entrepreneurs default. This formulation also implies that all entrepreneurs choose the same level of leverage, leading to an aggregation result by which it is not necessary to keep track of the distribution of net worth among entrepreneurs (which is particularly suitable for quantitative analysis). Each period $t + 1$, the realization of ω is drawn from a distribution $F_{\omega,t}(\omega, \sigma_t)$, where σ_t is an exogenous shock to the cross-sectional dispersion of idiosyncratic shocks.

On the other side of the market, it is assumed that financial intermediaries obtain funds by issuing one-period, non-state-contingent bonds, purchased by households (similar to deposits). Financial intermediaries are diversified across idiosyncratic shocks and have free entry.

Physical capital markets. As in Section 1.3, trade of unmatched capital between entrepreneurs and households occurs in a decentralized market with search frictions. In addition, this section also includes two centralized markets in which matched capital can be traded between entrepreneurs at price Q^c , and unmatched capital can be traded between households, financial intermediaries and retired entrepreneurs at price J^u . Including these two markets is convenient for technical reasons. In particular, the centralized market in which matched capital can be traded between entrepreneurs allows the analysis to focus on an equilibrium that does not depend on the distribution of capital among entrepreneurs.⁹ The centralized market in which unmatched capital can be traded facilitates the study of financial intermediaries, who, in the event of default, seize the entrepreneur's capital (recall that the entrepreneur's capital becomes unmatched in the event of default). Figure 1.7 sum-

⁹A key assumption in Bernanke, Gertler and Gilchrist (1999) for the result that all entrepreneurs choose the same level of leverage is the existence of a market in which entrepreneurs can trade physical capital. This aggregation result, which is particularly convenient for quantitative analysis, can be extended to the framework of the present paper if entrepreneurs are allowed to trade matched capital in a centralized market. Studying an economy in which a centralized market for trading matched capital does not exist is left for future research.

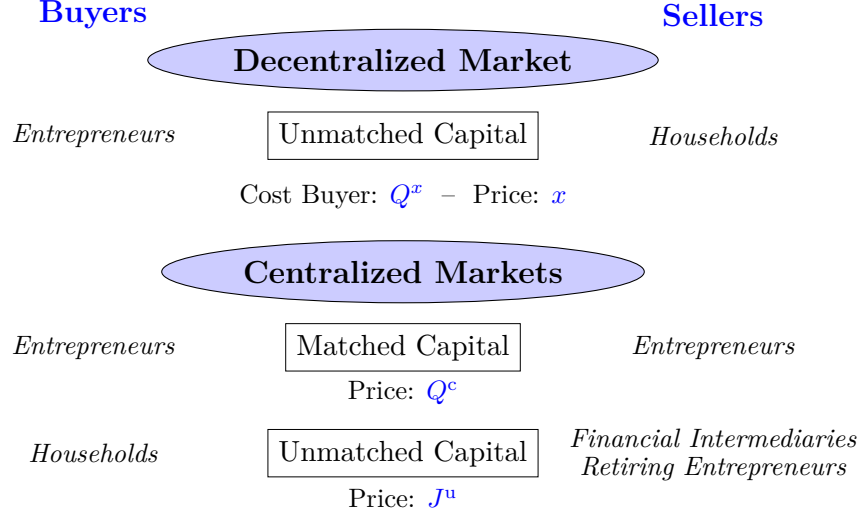


FIGURE 1.7: Structure of Capital Markets, Quantitative Model.

marizes these three markets for capital, with the participants and forms of trade that characterize each market.

Search frictions that characterize the decentralized market for unmatched capital are identical to those in Section 1.3. In particular, search is directed: The market is organized in a continuum of submarkets indexed by the price of unmatched capital, denoted x , and sellers (households) and buyers (entrepreneurs) can choose which submarket to visit. In each submarket, the market tightness, denoted $\theta(x)$ is defined as the ratio between the mass of capital searched by entrepreneurs and the mass of unemployed capital offered in that submarket. Households face no search cost. Visiting submarket x , they face a probability $p(\theta(x))$ of finding a match, where $p : \mathbb{R}_+ \rightarrow [0, 1]$ is a twice continuously differentiable, strictly increasing, strictly concave function that satisfies $p(0) = 0$ and $\lim_{\theta \rightarrow \infty} p(\theta) = 1$. Entrepreneurs face a cost per unit searched, $c_s > 0$, denoted in terms of consumption goods. Visiting submarket x , they face a probability $q(\theta(x))$ of finding a match, where $q : \mathbb{R}_+ \rightarrow [0, 1]$ is a twice continuously differentiable, strictly decreasing function that satisfies $q(\theta) = \frac{p(\theta)}{\theta}$, $q(0) = 1$ and $\lim_{\theta \rightarrow \infty} q(\theta) = 0$. The cost of a unit of capital for entrepreneurs in submarket x is denoted Q^x (which includes two components: the price paid to the

seller, x , and the search cost in submarket x).

Timing. Time is discrete and infinite, with each period divided into six stages: production, repayment, separation, borrowing, search and investment. In the *production stage*, entrepreneurs produce consumption goods using capital matched in the previous period. In the *repayment stage*, entrepreneurs repay their loans from the previous period or default; in case of default their capital becomes unmatched and financial intermediaries monitor and seize the entrepreneur's production and capital. In the *separation stage*, a fraction $\bar{\psi}$ of entrepreneurs that have not defaulted retires and their capital becomes unmatched. A new mass of entrepreneurs begins entrepreneurial activity with no initial capital and with an exogenously determined net worth. In the *borrowing stage*, entrepreneurs who do not retire and new entrepreneurs borrow from financial intermediaries, and financial intermediaries sell bonds to households. In the *search stage*, the remaining entrepreneurs purchase unmatched capital from households and matched capital from other entrepreneurs. In the *investment stage*, households produce unmatched physical capital and consume; retired entrepreneurs transfer their net worth, including unmatched capital, to their households; and financial intermediaries sell seized unmatched capital to households.

1.4.2 Households

Household preferences are described by the lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \{U(C_{i,t} - \rho_c C_{i,t-1}) - V(h_{i,t}; \varphi_t)\}, \quad (1.30)$$

where $C_{i,t}$ denotes consumption of household i in period t , $h_{i,t}$ denotes hours worked by household i in period t ; $\beta \in (0, 1)$ is the subjective discount factor; $\rho_c \in [0, 1)$ is a parameter governing the degree of internal habit formation; φ_t denotes an exogenous and stochastic preference shock in period t (labeled a *labor-wedge shock*); for every realization of φ_t , $V(\cdot; \varphi_t) : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a twice continuously differentiable, strictly

increasing, strictly convex function; $U : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a twice continuously differentiable, strictly increasing, strictly concave function; and E_t denotes the expectation conditional on the information set available at time t .

The stock of unemployed capital held by household i evolves according to

$$K_{i,t+1}^u = \int_0^{(1-\delta)K_{i,t}^u} (1 - p(\theta_t(x_{k,i,t}))) dk + \iota_{i,t}^h + A_t^I \left[I_{i,t} - \Phi \left(\frac{I_{i,t}}{K_t} \right) K_t \right], \quad (1.31)$$

where $K_{i,t}^u$ denotes the stock of unemployed capital held by household i at the beginning of period t , $x_{k,i,t}$ denotes the submarket in which unemployed capital unit k is listed by household i in period t , $\iota_{i,t}^h$ denotes the units of unmatched capital purchased by households in the centralized market in period t , $I_{i,t}$ denotes investment by household i in period t , K_t denotes aggregate capital stock at the beginning of period t (taken as given by household i), A_t^I denotes an exogenous aggregate shock that affects the production of capital from investment goods in period t (as in [Justiniano, Primiceri and Tambalotti, 2011](#), labeled an *investment-specific technology shock*), and $\Phi : \mathbb{R} \rightarrow \mathbb{R}$ is a twice continuously differentiable, strictly convex function that introduces investment-adjustment costs. The first term of the right-hand side of equation (1.31) represents the depreciated mass of capital unemployed at the beginning of period t and not sold to entrepreneurs for a given market-tightness function $\theta_t(x)$ and choice of submarket $x_{k,i,t}$. The second term of the right-hand side of equation (1.31) represents the mass of employed capital purchased by the households from retired and defaulting entrepreneurs. The third term represents the addition (subtraction) to unemployed-capital stock from investment, net of adjustment costs.

Households have access to a one-period, non-state-contingent bond issued by financial intermediaries. The household's sequential budget constraint is given by

$$C_{i,t} + I_{i,t} + J_t^u \iota_{i,t}^h + T_t + B_{i,t} = R_{t-1} B_{i,t-1} + W_t h_{i,t} + \int_0^{(1-\delta)K_{i,t}^u} p(\theta_t(x_{k,i,t})) x_{k,i,t} dk + \Pi_t, \quad (1.32)$$

where $B_{i,t}$ denotes the one-period bond holdings chosen by household i at the be-

ginning of period t , which pays a gross non-state-contingent interest rate, R_t ; W_t denotes the wage rate; Π_t denotes net transfers from entrepreneurs and financial intermediaries to households in period t – described further in the next sections; and T_t represents a lump-sum government tax (subsidy) in period t .

Household i 's problem is then to choose the state-contingent sequences of $C_{i,t}$, $h_{i,t}$, $I_{i,t}$, $\iota_{i,t}^h$, $K_{i,t+1}^u$, $B_{i,t}$ and $x_{k,i,t}$ that maximize the expected utility (1.30), subject to the sequence of budget constraints (1.32) and the accumulation constraints for unemployed capital (1.31), for the given initial levels of capital and consumption ($K_{i,0}^u$, K_0^e , K_0 , and $C_{i,-1}$), the given sequence of prices (W_t , J_t^u and R_t), the given sequence of dividends and taxes (Π_t and T_t), the given sequence of market-tightness functions ($\theta_t(x)$), and the given sequence of labor wedges (φ_t) and investment-specific productivities (A_t^I). Denoting the Lagrange multiplier associated with the budget constraint (1.32) as $\Lambda_{i,t}$, the optimality conditions in an interior solution are (1.32), (1.31), and the first-order conditions:

$$\Lambda_{i,t} = U'(C_{i,t} - \rho_c C_{i,t-1}) - \beta \rho_c E_t U'(C_{i,t+1} - \rho_c C_{i,t}), \quad (1.33)$$

$$\Lambda_{i,t} J_t^u = \beta \Lambda_{i,t+1} (1 - \delta) [p(\theta_{t+1}(x_{i,t+1}^u)) x_{i,t+1}^u - (1 - p(\theta_{t+1}(x_{i,t+1}^u))) J_{t+1}^u], \quad (1.34)$$

$$1 = J_t^u A_t^I [1 - \Phi' \left(\frac{I_{i,t}}{K_t} \right)], \quad (1.35)$$

$$\Lambda_{i,t} = \beta R_t E_t \Lambda_{i,t+1}, \quad (1.36)$$

$$-p(\theta(x_{i,t}^u)) = p'(\theta(x_{i,t}^u)) \theta'_t(x_{i,t}^u) (x_{i,t}^u - J_t^u), \quad (1.37)$$

$$V'(h_{i,t}; \varphi_t) = \Lambda_{i,t} W_t, \quad (1.38)$$

where $x_{i,t}^u$ denotes household i 's choice of submarket for unmatched capital in period t , and the unit of capital subindex, k , has been dropped because the optimality condition with respect to the choice of submarket, $x_{k,i,t}$, is the same for all units of capital.

1.4.3 Financial Intermediaries

Financial intermediaries sell one-period non-state-contingent bonds to households and lend to entrepreneurs. The set of contracts offered to entrepreneur j specifies an aggregate, state-contingent interest rate, $Z_{j,t+1}$, for each loan amount, $D_{j,t+1}$, to be repaid in case of no default. In case of default, the financial intermediary seizes the entrepreneur's assets, with a recovery value of $(1 - \mu_m)\mathcal{R}_{j,t+1}(\omega)$. Debt schedules available for entrepreneur j include all contracts $(Z_{j,t+1}, D_{j,t+1})$ that allow a financial intermediary to repay in all states the risk-free bond sold to households, after diversifying idiosyncratic risk:¹⁰

$$D_{j,t+1}R_t = [1 - F_\omega(\bar{\omega}_{j,t+1}; \sigma_t)]Z_{j,t+1}D_{j,t+1} + (1 - \mu_m) \int_0^{\bar{\omega}_{j,t+1}} \mathcal{R}_{j,t+1}(\omega) dF_\omega(\omega; \sigma_t), \quad (1.39)$$

where $\bar{\omega}_{j,t+1}$ denotes the default threshold in period $t + 1$ for entrepreneur j with outstanding debt $D_{j,t+1}$ and stock of matched capital $K_{j,t+1}^e$ – to be discussed in detail in the next section. The left-hand side of equation (1.39) represents the obligations assumed by the financial intermediary selling the risk-free bond to households. The right-hand side of equation (1.39) represents the resources obtained by the financial intermediary from lending, after diversifying over idiosyncratic risk. It includes two terms, representing resources from entrepreneurs who do not default and resources from those who do.

It is assumed that in the default state financial intermediaries monitor and seize the entrepreneur's production and capital. Hence,

$$\mathcal{R}_{j,t+1}(\omega) = [r_{j,t+1}^k + (1 - \delta)J_{t+1}^u]\omega K_{j,t+1}^e, \quad (1.40)$$

where $r_{j,t+1}^k$ denotes the net revenues from production per unit of effective capital, $\omega K_{j,t+1}^e$ – to be described in detail in the next section.

¹⁰For formulations of debt contracts similar to the one presented in this section, see [Arellano, Bai and Zhang \(2012\)](#) and [Christiano, Motto and Rostagno \(2014\)](#).

1.4.4 Entrepreneurs

Entrepreneurs have access to technology to produce consumption goods using labor and matched capital as inputs. In particular, the output produced by an effective unit of matched capital, ℓ , employing $\tilde{h}_{\ell,t}$ hours of work, is given by¹¹

$$y_{\ell,t} = A_t \left(\tilde{h}_{\ell,t} \right)^{1-\alpha}, \quad (1.41)$$

where $y_{\ell,t}$ denotes output in units of matched capital ℓ in period t and A_t is an exogenous aggregate productivity shock affecting the production technology in period t (labeled the *neutral-technology shock*).

Each period, entrepreneurs face an i.i.d. shock to the quality of their matched capital, denoted ω , drawn from a log-normal distribution with c.d.f. $F_\omega(\omega; \sigma_t)$ and satisfying $E_t(\omega_{t+1}) = 1 \forall t$ and $\text{Var}_t(\log(\omega_{t+1})) = \sigma_t^2 \forall t$, where σ_t is an exogenous aggregate shock to the cross-sectional dispersion of idiosyncratic shocks (labeled the *risk shock*, as in [Christiano, Motto and Rostagno, 2014](#)). Output produced by entrepreneur j with a mass of matched capital $K_{j,t}^e$, with $\tilde{h}_{j,t}$ hours worked in each of these units of capital and a utilization rate of $u_{j,t}$, denoted $Y_{j,t}$, is then given by

$$Y_{j,t} = A_t \left(\tilde{h}_{j,t} \right)^{1-\alpha} u_{j,t} \omega_{j,t} K_{j,t}^e, \quad (1.42)$$

where $\omega_{j,t}$ denotes the realization of the exogenous and stochastic variable ω for entrepreneur j in period t . The term $\omega_{j,t} K_{j,t}^e$ denotes the effective mass of matched capital held by entrepreneur j at the beginning of period t .

Entrepreneurs pay wage rate W_t per hour worked and face convex costs on the utilization rate. It follows that net revenues from production per unit of effective matched capital for entrepreneur j are given by

$$r_{j,t}^k = \left(A_t \left(\tilde{h}_{j,t} \right)^{1-\alpha} - W_t \tilde{h}_{j,t} \right) u_{j,t} - \mathcal{C}_u(u_{j,t}), \quad (1.43)$$

¹¹This production technology is similar to one in which production is carried out in a continuum of plants, as, for example, in [Cooley, Hansen and Prescott \(1995\)](#). In this framework, it can be shown that the aggregate production function of the economy displays constant returns to scale.

where $\mathcal{C}_u(u) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a twice continuously differentiable, strictly increasing, strictly convex function. Note that $r_{j,t}^k$ is independent of the mass of matched capital held by entrepreneur j , $K_{j,t}^e$, and independent of the realization of the idiosyncratic shock for entrepreneur j , $\omega_{j,t}$.

In this setup, all entrepreneurs face an expected linear rate of return per unit of capital purchased:

$$R_{j,t+1}^{k,m} \equiv \frac{r_{j,t+1}^k + (1 - \delta) [\bar{\psi} J_{t+1}^u + (1 - \bar{\psi}) Q_{t+1}^c]}{Q_t^m}, \quad (1.44)$$

for $m \in \{x, c\}$. The denominator of the right-hand side of (1.44) represents the price at which the effective unit of matched capital was purchased. The numerator of the right-hand side of (1.44) represents the sources of revenue per unit of effective matched capital. The first component of the numerator represents net revenue from production. The second component represents the expected revenue from selling the depreciated unit of effective matched capital. If the entrepreneur retires (with probability $\bar{\psi}$), this effective unit of matched capital is traded unmatched at a price J_{t+1}^u . If the entrepreneur does not retire (with probability $1 - \bar{\psi}$), this effective unit of matched capital is traded matched at Q_{t+1}^c .

Entrepreneurs purchase capital using their net worth and borrowing from financial intermediaries. This means that, at the end of each period t and for any entrepreneur j , the entrepreneur's balance sheet follows

$$\int_x Q_t^x \tilde{K}_{j,t+1}^x dx + Q_t^c \tilde{K}_{j,t+1}^c = D_{j,t+1} + N_{j,t+1}, \quad (1.45)$$

where $D_{j,t+1} \geq 0$ denotes debt contracted by entrepreneur j in period t , to be paid in period $t + 1$, $N_{j,t+1} \geq 0$ denotes the net worth of entrepreneur j at the end of period t , $\tilde{K}_{j,t+1}^x \geq 0$ denotes the stock of capital held by entrepreneur j at the end of period t , purchased in submarket x of the decentralized market at a cost per unit Q_t^x , and $\tilde{K}_{j,t+1}^c \geq 0$ denotes the stock of capital held by entrepreneur j at the end of period t purchased in the centralized market at a cost Q_t^c per unit. The latter case also

includes the stock of capital held by entrepreneur j from the previous period, which is equivalent to selling and repurchasing the unit in the centralized market at price Q_t^c . Note that $\int_x \tilde{K}_{j,t+1}^x dx + \tilde{K}_{j,t+1}^c = K_{j,t+1}^e$. The left-hand side of equation (1.45) represents the entrepreneur's assets, given by the value of the matched capital. The right-hand side of equation (1.45) represents the entrepreneur's liabilities and equity, given by debt with financial intermediaries and net worth.

As in Section 1.3, by the law of large numbers, the cost per unit of capital of mass \tilde{K}_{t+1}^x purchased in the submarket x of the decentralized market is given by

$$Q_t^x = x + \frac{c_s}{q(\theta_t(x))}. \quad (1.46)$$

The right-hand side of equation (1.46) represents the two components of the cost of a unit of capital in the decentralized market: the price paid to the seller, x , and the search cost, $\frac{c_s}{q(\theta_t(x))}$.

To solve the entrepreneur's problem, it is useful to define the entrepreneur's leverage and "portfolio weights," from the components of the entrepreneur's balance sheet (1.45). Leverage for entrepreneur j at the end of period t is defined by

$$L_{j,t} \equiv \frac{\int_x Q_t^x \tilde{K}_{j,t+1}^x dx + Q_t^c \tilde{K}_{j,t+1}^c}{N_{j,t+1}}. \quad (1.47)$$

The portfolio weight of each asset considered in the left-hand side of equation (1.45) is

$$w_{j,t}^m \equiv \frac{Q_t^m \tilde{K}_{j,t+1}^m}{L_{j,t} N_{j,t+1}}, \quad (1.48)$$

for $m \in \{x, c\}$. From (1.45) and the nonnegativity constraint of capital holdings ($\tilde{K}_{j,t+1}^x \geq 0$ for $m \in \{x, c\}$), it follows that $w_{j,t}^m \in [0, 1] \forall m \in \{x, c\}$.

Entrepreneurs are risk neutral and their objective is to maximize their expected

net worth, given at the end of period t by¹²

$$E_t \left\{ \int_{\bar{\omega}_{j,t+1}}^{\infty} \left[\omega \tilde{R}_{j,t+1}^k L_{j,t} N_{j,t+1} - Z_{j,t+1} D_{j,t+1} \right] dF_{\omega}(\omega; \sigma_t) \right\}, \quad (1.49)$$

where the portfolio return, denoted $\tilde{R}_{j,t+1}^k$, is defined by $\tilde{R}_{j,t+1}^k \equiv \int_x w_{j,t}^x R_{j,t+1}^{k,x} dx + w_{j,t}^c R_{j,t+1}^{k,c}$. The first term in the objective function (1.49) represents the revenue that will be received in period $t+1$ by entrepreneur j . The second term represents debt repayments to financial intermediaries. Given that the entrepreneur receives revenue and performs debt repayment only in case of not defaulting, these terms are integrated over the realizations of $\omega_{j,t}$ above $\bar{\omega}_{j,t+1}$.

From the objective function (1.49), it follows that the expected value for entrepreneur j of repaying debt $D_{j,t+1}$ in the repayment stage of period $t+1$ is given by

$$V_{j,t+1}^R = \omega_{j,t+1} \tilde{R}_{j,t+1}^k L_{j,t} N_{j,t+1} - Z_{j,t+1} D_{j,t+1}. \quad (1.50)$$

Given that the expected value of defaulting is equal to zero, equation (1.50) implies that the optimal default threshold, $\bar{\omega}_{j,t+1}$, is implicitly defined by

$$\bar{\omega}_{j,t+1} \tilde{R}_{j,t+1}^k L_{j,t} N_{j,t+1} = Z_{j,t+1} D_{j,t+1}. \quad (1.51)$$

Using (1.47) and (1.51) in (1.49), entrepreneur j 's objective function can be reexpressed as

$$E_t \left\{ \left[\int_{\bar{\omega}_{j,t+1}}^{\infty} \omega dF_{\omega}(\omega; \sigma_t) - (1 - F_{\omega}(\bar{\omega}_{j,t+1}; \sigma_t)) \bar{\omega}_{j,t+1} \right] \tilde{R}_{j,t+1}^k L_{j,t} N_{j,t+1} \right\}, \quad (1.52)$$

which is proportional to net worth $N_{j,t+1}$.

Similarly, substituting (1.51) and (1.47) into (1.39) and (1.40), the financial intermediaries' participation constraint is

$$\frac{L_{j,t} - 1}{L_{j,t}} R_t = [1 - F_{\omega}(\bar{\omega}_{j,t+1}; \sigma_t)] \bar{\omega}_{j,t+1} \tilde{R}_{j,t+1}^k + (1 - \mu_m) \int_0^{\omega_{j,t+1}} \omega dF_{\omega}(\omega; \sigma_t) \tilde{R}_{j,t+1}^{k,\psi}, \quad (1.53)$$

¹²The assumption that entrepreneurs are risk neutral maximize their expected net worth follows the quantitative literature implementing the costly state-verification framework. For a recent study relaxing this and other assumptions of the standard implementation of costly state verification used in this paper, see [Dmitriev and Hoddenbagh \(2013\)](#).

where the portfolio return conditional on separation is defined by $\tilde{R}_{j,t+1}^{k,\psi} \equiv \int_x w_{j,t}^x R_{j,t+1}^{k,\psi,x} dx + w_{j,t}^c R_{j,t+1}^{k,\psi,c}$, and $R_{j,t+1}^{k,\psi,m}$ denotes the return of an effective unit of separated capital, which, similar to (1.44), is defined by

$$R_{j,t+1}^{k,\psi,m} \equiv \frac{r_{j,t+1}^k + (1 - \delta)J_{t+1}^u}{Q_t^m}, \quad (1.54)$$

for $m \in \{x, c\}$. The combinations $(\bar{\omega}_{j,t+1}, L_{j,t}, \tilde{h}_{j,t}, u_{j,t}, w_{j,t}^x, w_{j,t}^c)$ that satisfy (1.53) define a menu of $(t+1)$ -contingent debt contracts offered to entrepreneurs equivalent to those defined in (1.39). Let $\mathcal{D}_t(\tilde{h}_{j,t}, u_{j,t}, w_{j,t}^x, w_{j,t}^c)$ denote the set of debt schedules $(\bar{\omega}_{j,t+1}, L_{j,t})$ offered to entrepreneurs by financial intermediaries.

Entrepreneur j 's problem is to choose the state-contingent plans $\tilde{h}_{j,t}$, $u_{j,t}$, $L_{j,t}$ and $\omega_{j,t+1}$, $w_{j,t}^x$, and $w_{j,t}^c$, with $(L_{j,t}, \omega_{j,t+1}) \in \mathcal{D}_t(\tilde{h}_{j,t}, u_{j,t}, w_{j,t}^x, w_{j,t}^c)$ that maximize the expected net worth (1.52) subject to the sequence of technological constraints, (1.43), return constraints, (1.44) and (1.54), and nonnegativity constraint for portfolio weights ($w_{j,t}^m \geq 0$ for $m \in \{x, c\}$) for the given sequence of prices (W_t , Q_t^c and J_t^u), debt schedules ($\mathcal{D}_t(\tilde{h}_{j,t}, u_{j,t}, w_{j,t}^x, w_{j,t}^c)$), market-tightness functions ($\theta_t(x)$), risk (σ_t), and neutral-technology shocks (A_t). With $\Lambda_{j,t+1}^e$ as the Lagrange multiplier on the financial intermediary's participation constraint, and $\Xi_{j,t}^m$ as the Lagrange multiplier associated with nonnegativity constraint for portfolio weights ($w_{j,t}^m \geq 0$), the optimality conditions are (1.43), (1.44), (1.53), (1.54), and

$$A_t(1 - \alpha) \left(\tilde{h}_t \right)^{-\alpha} = W_t, \quad (1.55)$$

$$\alpha A_t \tilde{h}_t^{(1-\alpha)} = C'_u(u_t), \quad (1.56)$$

$$E_t \left\{ [1 - \Gamma_t(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} - \Lambda_{t+1}^e \left[\frac{R_{t+1}^k}{R_t} \left(\Gamma_t(\bar{\omega}_{t+1}) - g_t(\bar{\omega}_{t+1}) + (1 - \mu_m)g_t(\bar{\omega}_{t+1}) \frac{R_{t+1}^{k,\psi}}{R_{t+1}^k} \right) - 1 \right] \right\} = 0, \quad (1.57)$$

$$\Lambda_{t+1}^e = \frac{\Gamma'_t(\bar{\omega}_{t+1})}{\Gamma'_t(\bar{\omega}_{t+1}) - \mu g'_t(\bar{\omega}_{t+1}) + (1 - \mu_m)g'_t(\bar{\omega}_{t+1}) \left(\frac{R_{t+1}^{k,\psi}}{R_{t+1}^k} - 1 \right)}, \quad (1.58)$$

$$Q_t^m = Q_t + \Xi_t^m, \quad (1.59)$$

and the complementary slackness conditions

$$\Xi_t^m \geq 0, \quad w_t^m \Xi_t^m = 0, \quad \text{for } m \in \{x, c\}, \quad (1.60)$$

where $\Gamma_t(\bar{\omega}_{t+1}) \equiv [1 - F_\omega(\bar{\omega}_{t+1}; \sigma_t)]\bar{\omega}_{t+1} + g_t(\bar{\omega}_{t+1})$, $g_t(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega dF_\omega(\omega; \sigma_t)$, and $Q_t \equiv \frac{r_{t+1}^k + (1-\delta)[\bar{\psi}J_{t+1}^u + (1-\bar{\psi})Q_{t+1}^c]}{R_{t+1}^k}$. The entrepreneur's subindex, j , has been dropped because the objective function is linear in the net worth of entrepreneur j and does not appear in any of the constraints. Therefore, all entrepreneurs will choose the same plans $(h_t, u_t, L_t \text{ and } \omega_{t+1})$, independent of net worth.

1.4.5 Government

The government is assumed to consume a stochastic amount of consumption goods, financed each period by levying lump-sum taxes on households. The government budget constraint is given by

$$G_t = T_t, \quad (1.61)$$

where G_t is government spending in period t (labeled the *government-spending shock*).

1.4.6 Equilibrium

In equilibrium, all centralized markets clear. For the centralized market for unmatched capital, equilibrium then requires that

$$\int_0^1 \iota_{i,t}^h di = \psi_t(1 - \delta)K_t^e, \quad (1.62)$$

where $K_t^e \equiv \int_0^1 K_{j,t}^e dj$ denotes the aggregate stock of employed capital at the beginning of period t , and $\psi_t \equiv (1 - g_{t-1}(\bar{\omega}_t))\bar{\psi} + g_{t-1}(\bar{\omega}_t)$ denotes the total share of employed capital that was separated in period t as a result of entrepreneurs' retirement and default. The left-hand side of (1.62) represents households' purchases in the market for unmatched capital. The right-hand side of (1.62) represents the mass of capital sold in the market for unmatched capital, from retired entrepreneurs and

financial intermediaries that seized capital of defaulting entrepreneurs (see Figure 1.7).

Replacing (1.62) in (1.31) and using the law of large numbers and the fact that the choice of submarket, $x_{k,i,t}$, is the same for all units of capital, k , and all households, i , and the choice of investment, $I_{i,t}$ is the same for all households i , the law of motion for unemployed capital is

$$K_{t+1}^u = (1 - p(\theta_t(x_t^u)))(1 - \delta)K_t^u + \psi_t(1 - \delta)K_t^e + A_t^I \left[I_t - \Phi \left(\frac{I_t}{K_t} \right) K_t \right], \quad (1.63)$$

where $K_t^u \equiv \int_0^1 K_{i,t}^u di$ denotes the aggregate stock to unemployed capital at the beginning of period t , and $I_t = \int_0^1 I_{i,t} di$ denotes aggregate investment.

Given that matched capital is homogeneous, no arbitrage between centralized and decentralized markets of matched capital requires $Q_t = Q_t^c$. Moreover, entrepreneurs' optimality conditions (1.59) and (1.60) imply that, in equilibrium, any submarket visited by a positive number of entrepreneurs must have the same cost per unit of capital, and entrepreneurs will be indifferent among them. Formally, for all x ,

$$\theta_t(x) \left(x + \frac{c_s}{q(\theta_t(x))} - Q_t \right) = 0. \quad (1.64)$$

This condition determines the equilibrium market-tightness function: For all $x < Q_t$,

$$\theta_t(x) = q^{-1} \left(\frac{c_s}{Q_t - x} \right). \quad (1.65)$$

For all $x \geq Q_t$, $\theta_t(x) = 0$.

Using the definition of market tightness, the law of large numbers, and the fact that a household's choice of submarket, $x_{k,t}$ is the same for all units of capital k , the flow of capital that transitions from unemployment to employment is given by $p(\theta_t(x_t^u))(1 - \delta)K_t^u = \int_0^1 \tilde{K}_{j,t+1}^{x^u} dj = \int_0^1 \int_x \tilde{K}_{j,t+1}^x dx dj$. Aggregating the entrepreneurs' capital-accumulation constraints and imposing market clearing in the centralized market provides a law of motion for employed capital:

$$K_{t+1}^e = (1 - \psi_t)(1 - \delta)K_t^e + p(\theta_t(x_t^u))(1 - \delta)K_t^u. \quad (1.66)$$

The capital-unemployment rate at the beginning of period t can be then defined as

$$k_t^u \equiv \frac{K_t^u}{K_t}, \quad (1.67)$$

where $K_t \equiv K_t^e + K_t^u$ denotes total aggregate capital stock at the beginning of period t .

Labor-market clearing requires $\int_0^1 \tilde{h}_{j,t} u_{j,t} \omega_{j,t} K_{j,t}^e dj = h_t$, where $h_t \equiv \int_0^1 h_t di$. Aggregating production functions (1.42) across entrepreneurs, using the fact that all entrepreneurs choose the same level of hours worked and utilization for each unit of effective capital and imposing the labor-market-clearing condition yields,

$$Y_t = A_t (u_t K_t^e)^\alpha (h_t)^{(1-\alpha)}, \quad (1.68)$$

where Y_t denotes aggregate output in period t .

Let ζ_t denote the exogenous aggregate net transfer from households to entrepreneurs in period t (labeled the *equity shock*). Aggregate net worth then evolves following the law of motion

$$N_{t+1} = (1 - \bar{\psi})[1 - \Gamma_{t-1}(\bar{\omega}_t)] R_t^{k,c} Q_{t-1} K_t^e + \zeta_t, \quad (1.69)$$

where N_{t+1} denotes aggregate net worth at the end of period t , and $R_t^{k,c}$ denotes the return of an effective unit of capital that does not separate in period t , which, similar to (1.44) and (1.54), is defined by $R_t^{k,c} \equiv \frac{r_t^k + (1-\delta)Q_t^c}{Q_{t-1}}$. The first term on the right-hand side of (1.69) represents the aggregate return obtained from effective matched capital employed in period t by entrepreneurs who did not default in the default stage and did not retire in the separation stage. The second term on the right-hand side of (1.69) represents the exogenous aggregate transfer from households to new entrepreneurs. The return obtained from effective matched capital employed in period t by entrepreneurs who did not default in the default stage, but did retire in the separation stage is transferred to households. It follows that the net transfer from

entrepreneurs to households is given by

$$\Pi_t = \bar{\psi}[1 - \Gamma_{t-1}(\bar{\omega}_t)]R_t^{k,\psi}Q_{t-1}K_t^e - \zeta_t. \quad (1.70)$$

where $R_t^{k,\psi} \equiv \frac{r_t^k + (1-\delta)J_t^u}{Q_{t-1}}$ denotes the return of an effective unit of capital that separates in period t .

Starting from the households' budget constraint (1.32) and replacing the government budget constraint (3.14), the market-clearing condition for unmatched capital (1.62), the market-clearing condition for the credit and labor markets, the definition of net revenues from production (1.43) and the participation constraints of financial intermediaries (1.39) aggregated across entrepreneurs, the expression for aggregate transfers from entrepreneurs (1.70) yields the economy's resource constraint,

$$C_t + I_t + G_t = Y_t - c_s\theta_t(1 - \delta)K_t^u - \Omega_t - C_u(u_t)K_t^e, \quad (1.71)$$

where $\Omega_t \equiv \mu g_{t-1}(\bar{\omega}_t)R_t^{k,\psi}Q_{k,t-1}K_t^e$ and aggregate consumption is defined by $C_t \equiv \int_0^1 C_{i,t} di$.

Let $S_t^x \equiv [A_t, A_t^I, G_t, \varphi_t, \sigma_t, \zeta_t]$ define the aggregate exogenous state vector of the economy. The competitive equilibrium in this economy can then be defined as follows.

Definition 3 (Competitive equilibrium). *Given initial conditions for employed and unemployed capital, K_0^e and K_0^u , consumption C_{-1} , and a state-contingent sequence of aggregate exogenous states, S_t^x , a competitive equilibrium is a state-contingent sequence of individual allocations and shadow values $\{(C_{i,t}, h_{i,t}, I_{i,t}, \iota_{i,t}^h, K_{i,t+1}^u, B_{i,t}, x_{i,t}^u)_{i \in [0,1]}, (\tilde{h}_{j,t}, u_{j,t}, L_{j,t}, \omega_{j,t+1}, w_{j,t}^x, w_{j,t}^c)_{j \in [0,1]}\}$, $\{(\Lambda_{i,t})_{i \in [0,1]}, (Q_{j,t})_{j \in [0,1]}\}$, aggregate allocations $\{C_t, I_t, h_t, K_{t+1}^e, K_{t+1}^u, N_t, \Pi_t\}$, prices $\{Q_t^c, J_t^u, W_t\}$, debt schedules $\{\mathcal{D}_t(\tilde{h}_{j,t}, u_{j,t}, w_{j,t}^x, w_{j,t}^c)\}$, and market-tightness functions $\{\theta_t(x)\}$, such that:*

- (i) *Individual allocations and shadow values solve the household's and entrepreneur's problems at the equilibrium prices, equilibrium market-tightness functions, and debt schedules, for all i and j .*

(ii) Debt schedules satisfy financial intermediaries' participation constraint (1.53).

(iii) The market-tightness function satisfies (1.65) for all x .

(iv) Centralized markets clear.

1.5 Quantitative Analysis

This section conducts a quantitative study of the role of search frictions in investment based on the model presented in Section 1.4. It begins by specifying assumptions for functional forms and stochastic processes contained in the model. It then discusses the empirical methodology for calibration and estimation of the model's parameters for the U.S. economy, presents estimation results, and conducts exercises based on the estimation related to the U.S. Great Recession and business cycles.

1.5.1 Model Estimation

Functional forms. The assumptions made on functional forms are standard in the related literature. For the households' period utility function,

$$\begin{aligned} U(c) &= \frac{c^{1-v}}{1-v} \\ V(h; \varphi) &= \varphi \frac{h^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}}, \end{aligned}$$

where $v > 0$ is the inverse of the intertemporal elasticity of substitution and $\phi > 0$ is the Frisch elasticity of labor supply.

Investment-adjustment costs are assumed to take a quadratic form:

$$\Phi\left(\frac{I_t}{K_t}\right) = \frac{\kappa}{2} \left(\frac{I_t}{K_t} - \delta\right)^2,$$

where $\kappa > 0$ is a parameter governing the degree of investment-adjustment costs.

Utilization costs are assumed to take the form

$$\mathcal{C}_u(u) = \alpha \bar{h}^{(1-\alpha)} \left[e^{c_u(u-1)} - 1 \right] \frac{1}{c_u},$$

where $c_u > 0$, and $\bar{\bar{h}}$ is the steady-state level of hours worked per unit of employed capital, defined by $\bar{\bar{h}} \equiv \frac{\bar{h}}{\bar{K}^e}$, where \bar{h} and \bar{K}^e are the steady-state level of hours worked and employed capital. As in [Christiano, Motto and Rostagno \(2014\)](#), this functional form is chosen to obtain a steady-state unity utilization rate independent of the parameter c_u .

The matching function is assumed to take a CES function, yielding the finding probabilities:

$$\begin{aligned} p(\theta) &= \theta (1 + \theta^\xi)^{-1/\xi}, \\ q(\theta) &= (1 + \theta^\xi)^{-1/\xi}, \end{aligned}$$

where $\xi > 0$. This functional form has been used in quantitative studies of directed search in the labor market (see, for example, [Schaal, 2012](#)).

Stochastic processes. The six aggregate shocks are modeled as first-order autoregressive processes:

$$\begin{aligned} \log A_t &= \rho_A \log A_{t-1} + \epsilon_t^A, \\ \log A_t^I &= \rho_{A^I} \log A_{t-1}^I + \epsilon_t^I, \\ \log G_t &= (1 - \rho_G) \log \bar{G} + \rho_G \log G_{t-1} + \epsilon_t^G, \\ \log \varphi_t &= (1 - \rho_\varphi) \log \bar{\varphi} + \rho_\varphi \log \varphi_{t-1} + \epsilon_t^\varphi, \\ \log \sigma_t &= (1 - \rho_\sigma) \log \bar{\sigma} + \rho_\sigma \log \sigma_{t-1} + \epsilon_t^\sigma, \\ \zeta_t &= (1 - \rho_\zeta) \bar{\zeta} + \rho_\zeta \zeta_{t-1} + \epsilon_t^\zeta, \end{aligned}$$

where $\bar{G} > 0$ denotes steady-state government spending, $\bar{\varphi} > 0$ is a parameter that determines steady-state hours worked, $\bar{\sigma} > 0$ denotes the steady-state cross-sectional dispersion of idiosyncratic shocks, $\bar{\zeta}$ denotes steady-state lump-sum transfers from households to entrepreneurs, and it is assumed that $\epsilon_t^i \sim N(0, \sigma^i) \forall t$ and $i \in \{A, A^I, G, \varphi, \sigma, \zeta\}$.

Data. The model is estimated using U.S. quarterly data prior to the Great Recession, from 1980:Q1 to 2007:Q4.¹³ The data include six time series: real per capita GDP, real per capita consumption, real per capita nonresidential private investment, per capita hours worked, credit spreads, and commercial, nonresidential real estate vacancy rates. Data on GDP, consumption and investment were log-linearly detrended. Credit spreads were measured by the difference between the interest rate on BAA corporate bonds and the three-month U.S. government bond rate. Appendix A.1 provides more detailed information about the sources and construction of these data.

Including data on GDP, consumption, investment, and hours is standard in the empirical business-cycle literature. Including credit spreads is relevant to discipline the financial friction and financial shocks (see Christiano, Motto and Rostagno, 2014). The counterpart of this variable in the model is the difference between the interest rate paid by entrepreneurs, Z_t , and the risk-free rate R_t . Including data on the commercial-real-estate vacancy rate (see Figure 1.2) is a novel feature of the present paper and is aimed at disciplining the search friction in investment – specifically, the two parameters related to search frictions, the curvature of the matching function, ξ , and the search cost, c_s . The counterpart of this variable in the model is the capital-unemployment rate, k_t^u .

It is assumed that all series are observed with measurement error. Measurement error in output, consumption, investment, hours worked, credit spreads and vacancy rates, denoted $\epsilon_{Y,t}^{\text{me}}$, $\epsilon_{C,t}^{\text{me}}$, $\epsilon_{I,t}^{\text{me}}$, $\epsilon_{h,t}^{\text{me}}$, $\epsilon_{s,t}^{\text{me}}$ and $\epsilon_{k^u,t}^{\text{me}}$, are assumed to be i.i.d. innovations with mean zero and standard deviation $\sigma_i^{\text{me}} \forall i \in \{Y, C, I, h, s, k^u\}$.

Empirical strategy. From the assumed functional forms and stochastic processes in the previous sections, the model features 27 structural parameters. Let Θ be a

¹³The estimation period begins in 1980 due to the availability of commercial-real-estate vacancy rates.

vector containing all the parameters of the model. This vector also includes the six nonstructural parameters representing the standard deviations of the measurement errors on the observables, as discussed in the previous section. The model parameters are partitioned into two sets: $\Theta = [\Theta_1, \Theta_2]$. The first set,

$$\Theta_1 \equiv [\beta, v, \phi, \alpha, \delta, \bar{\psi}, \mu_m, \bar{G}, \bar{\varphi}, \bar{\sigma}],$$

contains 10 calibrated or fixed a priori parameters. The remaining 23 parameters,

$$\Theta_2 \equiv [\rho_c, \kappa, c_u, \xi, c_s, \rho_A, \rho_{A^1}, \rho_G, \rho_\varphi, \rho_\sigma, \rho_\zeta, \sigma_A, \sigma_{A^1}, \sigma_G, \sigma_\varphi, \sigma_\sigma, \sigma_\zeta, \\ \sigma_Y^{\text{me}}, \sigma_C^{\text{me}}, \sigma_I^{\text{me}}, \sigma_h^{\text{me}}, \sigma_s^{\text{me}}, \sigma_{ku}^{\text{me}}],$$

are estimated using Bayesian methods surveyed in [An and Schorfheide \(2007\)](#). The following sections discuss the values assigned to parameters fixed a priori and the estimation of the remaining parameters.

Benchmark model without investment search frictions. To put the results of the estimated model from Section 1.4 into perspective, a benchmark model for the U.S. economy is also estimated. This benchmark model, detailed in Appendix A.5, is identical to the model of Section 1.4 except for the search friction in investment considered in this paper. The same empirical strategy described in the previous section is used for the benchmark model. The only differences are that the set of parameters Θ_2 does not include the parameters related to the search friction (i.e., ξ and c_s), and that the structure vacancy data are not included in the estimation as an observable. Henceforth, the model in Section 1.4 is labeled as the “Model with Search Frictions” and the benchmark model as “Model No Search Frictions.”

Calibrated parameters. Table 1.1 displays the values assigned to the calibrated parameters, contained in the vector Θ_1 or related targets. The subjective discount factor, β , the inverse of the intertemporal elasticity of substitution, v , the Frisch

TABLE 1.1
CALIBRATED PARAMETERS

Parameter	Value	Description
β	0.99	Subjective discount factor
ν	2	Intertemporal elasticity of substitution
ϕ	1	Frisch elasticity of labor supply
α	0.4	Capital share
δ	0.025	Depreciation rate of capital
ψ	0.028	Retirement rate of entrepreneurs
\bar{G}	0.2	Steady-state share of government spending
\bar{h}	1	Steady-state hours worked
\bar{s}	0.02	Steady-state annual spreads
$\bar{\mu}$	0.2	Steady-state loss in default
$F(\bar{\omega})$	0.075	Default rate

Note: The time unit is one quarter.

elasticity of labor supply, ϕ , the aggregate capital share, α , and the depreciation rate, δ , are set to 0.99, 2, 1, 0.4, and 0.025, respectively, standard values in related business cycle literature. The labor disutility parameter $\bar{\varphi}$ is set at a value consistent with a steady-state level of hours worked of one. The value of $\bar{\psi}$ is set to 0.027, which is consistent with the average annual exit rate of establishments in the United States for the period 1980–2007 of 11%. This value is also in line with the death rate of entrepreneurs in quantitative implementations of the costly state-verification framework. The value of the steady-state share of government spending, \bar{G} , was set at 0.2, a standard value in business-cycle studies for the U.S. economy.

The values used for the parameters related to the financial friction (μ_m , $\bar{\omega}$, and $\bar{\zeta}$) are close to those used in previous quantitative studies of the costly state verification. In particular, the values of $\bar{\omega}$ and $\bar{\zeta}$ and were set to target values of annual default rate and annual spreads of 3% and 200 basis points, respectively, which correspond to the U.S. historical averages (used for example in [Bernanke, Gertler and Gilchrist, 1999](#)). To set the value of the parameter μ_m , note that in the framework of the present paper, financial intermediaries in the state of default face a loss of the return of capital R_t^k not only related to monitoring costs (as in previous models with costly

state verification, but without search frictions in investment), but also related to the fact that capital becomes unmatched in the event of default (and has a return of $R_t^{k,\psi}$ instead of R_t^k ; see Sections 1.4.3 and 1.4.4). To make the loss in default comparable to those of previous studies – e.g., between 0.2 and 0.36 in Carlstrom and Fuerst (1997); 0.12 in Bernanke, Gertler and Gilchrist (1999) – μ_m was set to target a value of steady-state loss in default, $\bar{\mu}$, of 0.2, where the steady-state loss in default is defined as $\bar{\mu} \equiv 1 - (1 - \mu_m) \left(\frac{\bar{R}^{k,\psi}}{\bar{R}^k} \right)$, with $\bar{R}^{k,\psi}$ and \bar{R}^k denoting the steady-state values of $R_t^{k,\psi}$ and R_t^k .¹⁴

Estimated parameters. Table 1.2 presents the assumed prior distributions of the estimated parameters contained in the vector Θ_2 , denoted $P(\Theta_2)$. For the two parameters related to the search friction in investment – namely, the curvature of the matching function, ξ , and the search cost, c_s , for which, to my knowledge, estimates are not available – inverse gamma distributions were chosen. The mean of the distribution of the curvature of the matching function (ξ) was set at the value of 1. The mean of the distribution of the search cost parameter, c_s was set to 0.06 to target a steady-state level of capital under the mean of the prior distributions equal to the one observed in the data. For the other parameters, prior distributions were chosen following the related literature estimating models for the U.S. economy (Smets and Wouters, 2007; Schmitt-Grohé and Uribe, 2012a; Christiano, Motto and Rostagno, 2014).

In particular, the standard errors of the innovations are assumed to follow an inverse-gamma distribution with a mean of 0.1 and a standard deviation of 2; the persistence of the autoregressive stochastic processes, a beta distribution with mean 0.5 and standard deviation of 0.2; the parameter that governs internal habit formation (ρ_c), a beta distribution with mean 0.5 and standard deviation of 0.2; the parameter

¹⁴In the benchmark model without search frictions, $\mu_m = \bar{\mu}$.

that governs investment adjustment costs (κ), a gamma distribution with mean 3 and standard deviation of 2; and the parameter that governs the curvature of capital utilization costs (c_u), an inverse-gamma distribution with mean 2.5 and standard deviation of 2. Finally, uniform prior distributions were chosen for the innovations of the measurement error. These variables are restricted to account for at most 6% of the variance of the corresponding observable time series.

Given the prior parameter distribution, $P(\Theta_2)$, the Metropolis–Hastings algorithm was used to obtain draws from the posterior distribution of Θ_2 , denoted $\mathcal{L}(\Theta_2|Y)$ where Y is the data sample (see, for example, [An and Schorfheide, 2007](#)). Table

TABLE 1.2
ESTIMATED PARAMETERS ON U.S. DATA - MODEL WITH SEARCH FRICTIONS

Parameter	Description	Prior distribution			Posterior distribution	
		Distribution	Mean	St. dev	Mean	St. dev
A. Economic parameters						
ρ_c	Habit parameter	Beta	0.5	0.2	0.62	0.04
κ	Investment-adj costs	Gamma	3	2	4.3	0.3
ξ	Curvature-matching tech	Inv Gam	1	0.1	0.50	0.02
c_s	Search cost	Inv Gam	0.06	0.005	0.08	0.01
c_u	Curvature-utilization	Inv Gam	2.5	2	3.1	0.26
B. Stochastic processes						
Autocorrelations						
ρ_A	Neutral technology	Beta	0.5	0.2	0.98	0.01
ρ_{A^I}	Investment-specific tech	Beta	0.5	0.2	0.91	0.05
ρ_G	Government spending	Beta	0.5	0.2	0.91	0.02
ρ_φ	Labor wedge	Beta	0.5	0.2	0.986	0.004
ρ_σ	Risk	Beta	0.1	0.5	0.78	0.04
ρ_ζ	Equity	Beta	0.1	0.5	0.83	0.05
Standard deviation innovation						
σ_A	Neutral technology	Inv Gam	0.1	2	0.005	0.0004
σ_{A^I}	Investment-specific tech	Inv Gam	0.1	2	0.03	0.003
σ_G	Government spending	Inv Gam	0.1	2	0.02	0.002
σ_φ	Labor wedge	Inv Gam	0.1	2	0.02	0.002
σ_σ	Risk	Inv Gam	0.1	2	0.09	0.01
σ_ζ	Equity	Inv Gam	0.1	2	0.05	0.005

Note: The time unit is one quarter. Bayesian estimates are based on 500,000 draws from the posterior distribution.

TABLE 1.3
SECOND MOMENTS: DATA AND MODEL

	Standard deviations					
	σ_Y	$\frac{\sigma_C}{\sigma_Y}$	$\frac{\sigma_I}{\sigma_Y}$	$\frac{\sigma_h}{\sigma_Y}$	$\frac{\sigma_s}{\sigma_Y}$	$\frac{\sigma_{k^u}}{\sigma_Y}$
Data	4.1	0.82	2.80	1.39	0.45	0.55
Model with search	3.9	0.84	5.93	1.14	0.89	0.56
Model no search	2.8	0.99	4.02	1.08	0.45	
	Correlations with output					
	$\rho(C,Y)$	$\rho(I,Y)$	$\rho(h,Y)$	$\rho(s,Y)$	$\rho(k^u,Y)$	
Data	0.90	0.51	0.78	−0.53	−0.28	
Model with search	0.36	0.80	0.65	−0.40	−0.21	
Model no search	0.63	0.70	0.49	0.17		
	Autocorrelations					
	$\rho(Y_t, Y_{t-1})$	$\rho(C_t, C_{t-1})$	$\rho(I_t, I_{t-1})$	$\rho(h_t, h_{t-1})$	$\rho(s_t, s_{t-1})$	$\rho(k_t^u, k_{t-1}^u)$
Data	0.98	0.99	0.98	0.99	0.87	0.99
Model with search	0.94	0.99	0.89	0.92	0.84	0.98
Model no search	0.95	0.97	0.93	0.94	0.75	

Note: Columns labeled Y , C , I , h , s , and k^u refer, respectively, to output, consumption, investment, hours worked, credit spreads, and capital unemployment in the model. Data counterparts described in Appendix A.1. The time unit is one quarter. Data corresponds to the period 1962–2013, except for capital unemployment, which corresponds to the period 1980–2013.

1.2 presents the posterior estimates of the model parameters with search frictions in investment.

Model fit. The predictions of the model regarding standard deviations, correlation with output and serial correlations of the six time series included in the estimation as observables are presented in Table 1.3, together with their data counterparts. The predictions of the benchmark model without search frictions in investment are also presented in Table 1.3 for comparison.

Overall the predictions of the estimated models are in line with empirical second moments. The predicted standard deviations of the model with search frictions are in general larger than the one of the model without search frictions. For output, consumption, hours worked and capital unemployment the predictions of the model with search friction are similar to those observed in the data; for investment and

credit spreads the model with search frictions predicts a higher volatility than the one observed in the data. The correlations with output and autocorrelations predicted by the estimated models are in general in line with those observed in the data. For the case of credit spreads, while the estimated model without search frictions predicts a positive correlation with output, the model with search frictions in investment predicts a negative correlation with output, as observed in the data.

1.5.2 Quantitative Results

This section presents two exercises based on the estimated model to study the quantitative relevance of the proposed mechanism. The first relates to the Great Recession, which is an example of a deep financial crisis of the sort that motivated this theoretical framework (see Section 1.1). The second exercise studies the role of financial shocks in U.S. business cycle fluctuations in the presence of search frictions in investment. Both exercises proceed by comparing the results from the model presented in Section 1.4 to a benchmark model without investment search frictions, as presented in the previous section and detailed in Appendix A.5.

Recovery from the U.S. Great Recession. The estimated model is used to ask whether, following a sequence of shocks such as those experienced by the U.S. economy in 2008, and without any further shock, the model can predict an investment slump such as the one observed following the U.S. Great Recession – that, as discussed in Section 1.1, is an empirical regularity of financial-crisis episodes. To answer this question, the estimated model is used to smooth the shocks experienced by the U.S. economy through the last quarter of 2008. Beginning in the first quarter of 2009, the predicted response of the economy is computed: All shocks are set to zero, and the driving stochastic processes are only driven by their estimated autoregressive components; states evolve endogenously.

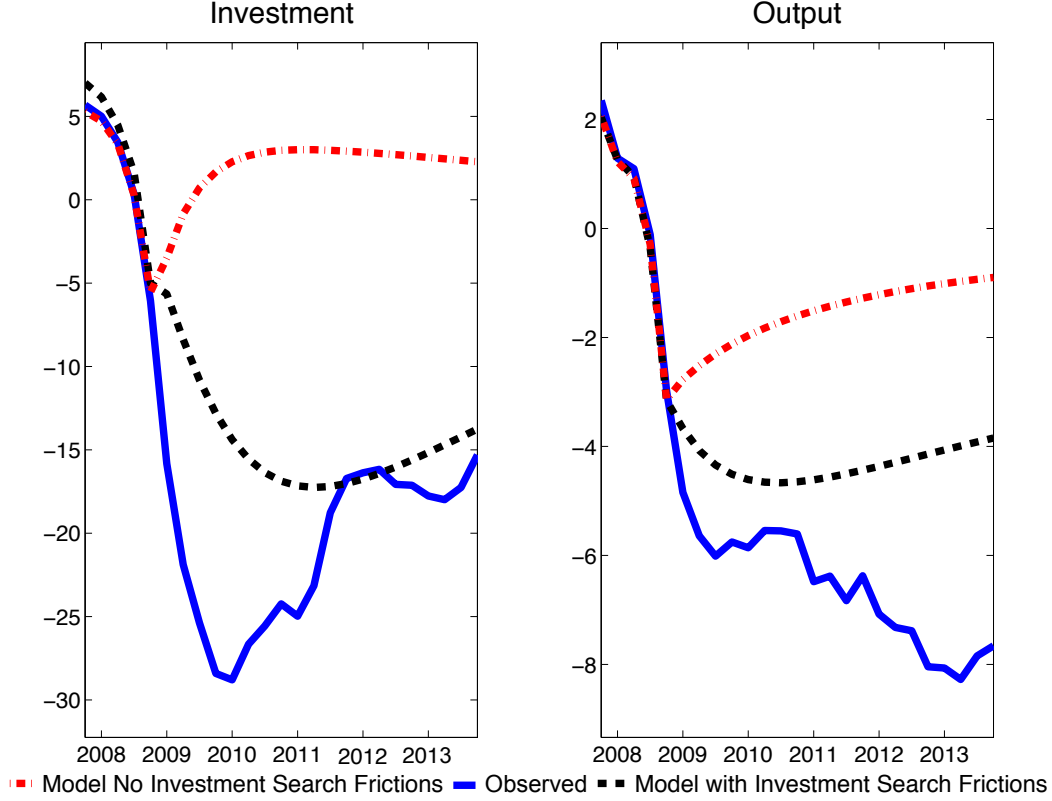


FIGURE 1.8: U.S. Great Recession: Predicted Recovery.

Note: Time-series labeled *Observed* correspond to the data on real per capital investment and output, log-linearly detrended (see Appendix A.1 for details). Time-series labeled *Model with Search Frictions* and *Model No Search Frictions* refer, respectively, to predictions from the model presented in Section 1.4, and to predictions from the benchmark model presented in Appendix A.5. Model predictions computed since 2009, following the sequence of shocks smoothed from the estimated models for the period 1980–2007. The time unit is one quarter. For details on the models’ estimations see Section 1.5.1 and Appendix A.5.

Results from this exercise are displayed in Figure 1.8 and indicate that the model with investment search frictions predicts a slump of investment following the U.S. Great Recession even larger than the one observed in the data. The same exercise in the benchmark model without investment search frictions predicts that both investment and output should be significantly higher than the levels observed in the data, as noted in the previous literature (see Section 1.1). The right panel of Figure 1.8 also shows that the proposed model with search frictions in investment can account for 50% of the difference between the observed recovery and the recovery predicted

by the benchmark model without search frictions.¹⁵

The Role of Financial Shocks in U.S. Business Cycles. The estimated model can also be used to interpret the sources of U.S. business-cycle fluctuations. Table 1.4 compares the variance decomposition predicted by the model with investment search frictions to the variance decomposition predicted by the benchmark model without search frictions. The most remarkable result is the difference between the two models in term of the contribution of financial shocks. The benchmark model without investment search frictions assigns a small role to financial shocks, and attributes most of the predicted movements in output and investment to technology shocks (neutral and investment-specific) and to labor wedge shocks. The model with search frictions developed in this paper attributes a relevant role to financial shocks, which account for 33% of output fluctuations and 56% of investment fluctuations.

This result is of interest since the role of financial shocks is a key discussion in the business-cycle literature and an important source of discrepancy between real and monetary models, with the latter attributing a much larger effect to these shocks than the former (as discussed in [Christiano, Motto and Rostagno, 2014](#)). The present paper shows that an important part of this discrepancy between these two branches of the literature can be reconciled by introducing investment search frictions. To understand this result, note that in the model with search frictions in investment, 63% of the predicted movements in capital unemployment are explained by financial shocks. Studying impulse-response functions, the next section comes back to this result.

¹⁵It is worth noting that the model's prediction for capital unemployment is in line the data on vacancy rates observed in the Great Recession. This variable and the prediction for the rest of the observables are included in Appendix A.2, showing that for all variables the model with search frictions in investment predict less recovery than the model without search frictions in investment.

TABLE 1.4
VARIANCE DECOMPOSITION

Shock		Y	C	I	h	s	k^u
Model no search							
Neutral technology	A	31.8	33.1	13.2	15.9	1.3	
Investment-specific technology	A^I	24.5	20.0	55.9	19.4	42.5	
Labor wedge	φ	42.0	45.3	15.8	60.0	1.6	
Government spending	G	0.8	1.1	4.6	3.2	0.6	
Risk	σ	0.1	0.1	3.3	0.4	44.7	
Equity	ζ	0.7	0.5	7.2	1.1	9.5	
Model with search							
Neutral technology	A	17.4	18.8	7.0	6.0	0.1	0.8
Investment-specific technology	A^I	4.1	5.4	23.5	11.6	5.6	34.6
Labor wedge	φ	44.6	56.9	11.9	54.8	0.2	1.3
Government spending	G	0.5	0.7	1.4	1.6	0.0	0.3
Risk	σ	0.5	0.3	2.0	0.4	39.1	15.0
Equity	ζ	33.0	17.9	54.2	25.6	55.0	47.9

Note: Columns labeled Y , C , I , h , s , and k^u refer, respectively, to output, consumption, investment, hours worked, credit spread, and capital unemployment in the model. Data counterparts described in Appendix A.1.

Impulse responses. To further study the quantitative findings presented in this section, Figure 1.9 shows the impulse response of capital unemployment, investment, and output to a one-standard-deviation negative neutral-technology shock and a one-standard-deviation negative equity shock. The responses of investment and output in the benchmark model without investment search frictions are also included for comparison.¹⁶ While a negative neutral-technology shock generates a decrease of capital unemployment, a negative equity shock generates an increase in capital unemployment. Moreover the response of capital unemployment is 10 times larger in absolute value in response to a one-standard-deviation equity shock than in response to a one-standard-deviation neutral-technology shock. For this reason, the responses of investment and output are more different in the case of the financial shock than in the case of a neutral-technology shock. The impulse-response functions also indicate a large and persistent effect on investment and output following a negative financial

¹⁶Standard deviations refer to those of the model with search frictions in investment. Appendix A.2 shows the impulse response for all shocks and for the six observables included in the estimation.

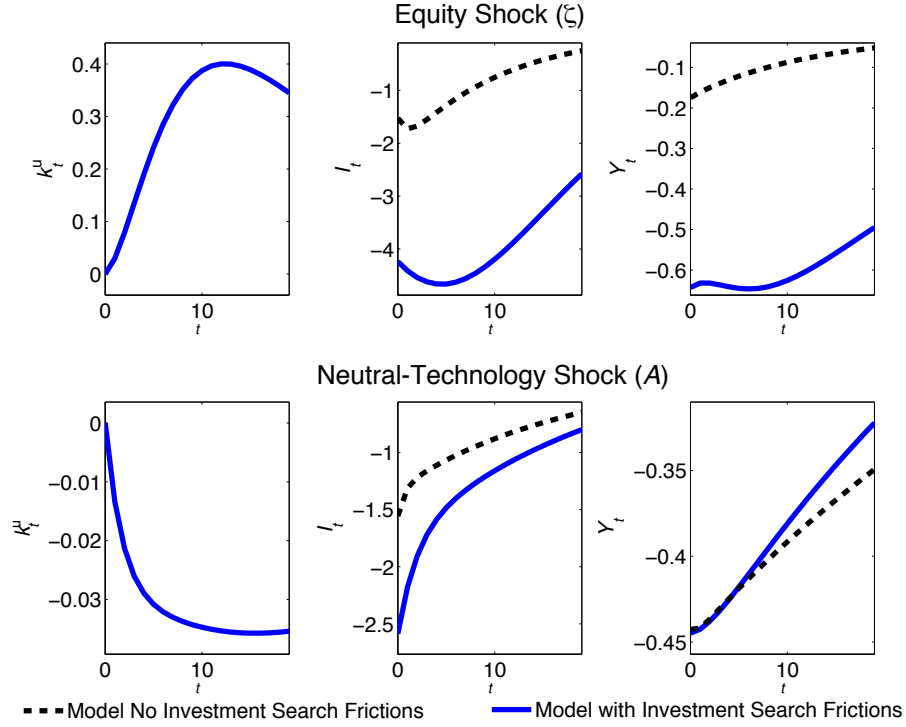


FIGURE 1.9: Impulse-Responses to Contractionary Shocks.

Note: Response of capital unemployment, investment, and output to a one-standard-deviation negative equity shock (ζ) and a neutral-technology shock (A). Label “Model with Search Frictions in Investment” and “Model No Search Frictions in Investment” refer, respectively, to the model responses presented in Section 1.4 and the benchmark model in Appendix A.5. Impulse responses expressed in percent deviations from steady state. Horizontal axes display quarters after the shock.

shock that is not present in the benchmark model without investment search frictions.

1.6 Capital Reallocation

This section shows that the analytical framework with investment search frictions developed in this paper can also be used to study capital reallocation. It begins by extending the model to allow for heterogeneity in capital match-specific productivity. This extension allows a characterization not only of the transition of capital from unemployment to employment, but of the transition of capital from employment to employment, since it adds a motive for trading capital while it remains employed

(similar to “on the job search” in the labor-market literature; see [Menzio and Shi, 2011](#)). A quantitative analysis of the extended model shows that the model’s predictions regarding capital reallocation are in line with those observed in the data. The model also has predictions regarding misallocation during crises.

1.6.1 Extended Model with Capital Reallocation

The basis of the analytical framework developed in this section is the quantitative model developed in [Section 1.4](#). The section begins by describing the extended model’s new assumptions regarding production technology and the market structure of physical capital. It then discusses the problem of selling employed capital in the decentralized market, the entrepreneur’s problem, and equilibrium in the extended framework. The notation used in this section is the same as that presented in [Section 1.4](#).

Production technology. As in [Section 1.4](#), it is assumed that entrepreneurs have access to technology to produce consumption goods using labor and matched capital as inputs. Unlike in [Section 1.4](#), each unit of employed capital has a match-specific productivity. This match-specific productivity is revealed after an unmatched unit of capital becomes matched, and does not vary until the specific match is destroyed. The output produced by an effective unit of capital i , with match-specific productivity z_i , and employing \tilde{h}_{it} hours of work, is given by

$$y_{i,t} = A_t z_{i,t} \left(\tilde{h}_{i,t} \right)^{1-\alpha}, \quad (1.72)$$

where $z_i \in \mathcal{Z} = \{z_1, z_2, \dots, z_{N_z}\}$, $N_z \geq 2$ and $\mathcal{Z} \gg 0$.

Physical capital markets. As in [Section 1.4](#), capital held by entrepreneurs is denoted employed capital, and capital held by households is denoted unemployed capital. Households can only hold unmatched capital. Trade of unmatched capital

between entrepreneurs (buyers) and households (sellers) occurs in a decentralized market with search frictions. The search frictions that characterize the decentralized market for unmatched capital are identical to those in Sections 1.3 and 1.4. Unlike in Section 1.4, entrepreneurs now also have access to the decentralized market as sellers, where they can sell an employed unit of capital as unmatched capital to other entrepreneurs. When a unit of capital employed with match-specific productivity z_i is traded in the decentralized market, a new match-specific productivity is drawn from the set \mathcal{Z} , with a probability mass function $f_Z(z) : \mathcal{Z} \rightarrow [0, 1]$, assumed to be the same for all t . Let \bar{z} denote the expected match-specific productivity of a new match (i.e. $\bar{z} \equiv E(z_i)$). It is assumed that $\bar{z} \in \mathcal{Z}$.

As in Section 1.4, entrepreneurs also have access to a centralized market in which they trade matched capital. When a unit of employed capital is traded in the centralized market it maintains its match-specific productivity. The match-specific productivity of any unit of capital is common knowledge. The difference with respect to Section 1.4 is that now units of capital matched at different match-specific productivities will be traded at different prices. The price in the centralized market of a unit of capital with match-specific productivity z_i is denoted Q^{z_i} . The price in the centralized market of a unit of capital matched at the average productivity \bar{z} will be denoted $Q^{\bar{z}}$.

Finally, as in Section 1.4 there is also a centralized market in which unmatched capital can be sold by financial intermediaries and retired entrepreneurs to households at price J^u . Figure 1.10 summarizes these three markets for capital, with the participants and forms of trade that characterize each market.

Seller's problem for employed capital. An entrepreneur that holds a unit of employed capital matched at productivity z_i can choose to sell this unit in the decentralized market – as an unmatched unit of capital – just as households do with

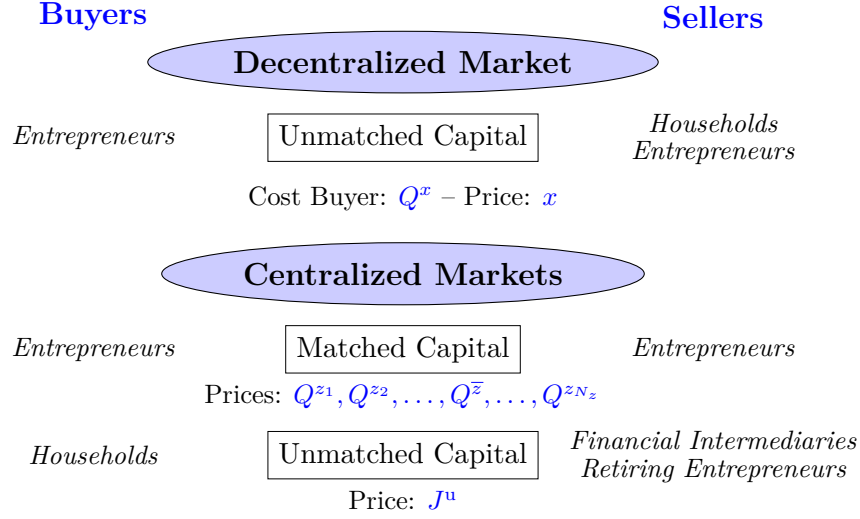


FIGURE 1.10: Structure of Capital Markets, Model with Capital Reallocation.

their units of unemployed capital. The only difference between entrepreneurs and households when visiting the decentralized market as sellers is that in the event of not finding a buyer the price of a unit of matched capital is different from the price of a unit of unmatched capital. Therefore, entrepreneurs who visit the decentralized market as sellers and households will typically search in different submarkets. For the same reason entrepreneurs holding units of capital at different match-specific productivities will also search in different submarkets. Formally, the *seller's problem* for an entrepreneur holding a unit of employed capital matched at productivity z_i is given by

$$\max_{x_t^{z_i}} \{p(\theta_t(x_t^{z_i})) x^{z_i} + (1 - p(\theta_t(x_t^{z_i}))) Q_t^{z_i}\}, \quad (1.73)$$

where $x_t^{z_i}$ denotes the submarket visited by an entrepreneur that holds a unit of capital matched at productivity z_i .

Entrepreneur's problem. As in Section 1.4, entrepreneurs purchase capital using their net worth and borrowing from financial intermediaries. Including match-specific productivity into the framework developed in Section 1.4 implies that the entrepreneur's balance sheet now includes different types of assets purchased in the cen-

tralized market. At the end of each period, t , equation (1.74) describes entrepreneur j 's balance sheet:

$$\int_x Q_t^x \tilde{K}_{j,t+1}^x dx + \sum_i Q_t^{z_i} \tilde{K}_{j,t+1}^{z_i} = D_{j,t+1} + N_{j,t+1}, \quad (1.74)$$

where $\tilde{K}_{j,t+1}^x$ denotes the stock of matched capital held by entrepreneur j at the end of period t , purchased in the submarket x of decentralized market, at a cost Q_t^x per unit of capital; and $\tilde{K}_{j,t+1}^{z_i}$ denotes the stock of capital matched with productivity z_i held by entrepreneur j at the end of period t purchased in the centralized market at price $Q_t^{z_i}$. The latter case also includes the stock of capital matched with productivity z_i held by entrepreneur j from the previous period, which is equivalent to selling and repurchasing the unit in the centralized market at price $Q_t^{z_i}$.

As in Section 1.4, to solve the entrepreneur's problem, it is useful to define the entrepreneur's leverage and "portfolio weights," from the components of the entrepreneurs balance sheet (1.74). The entrepreneur's leverage in period t is defined as

$$L_{j,t} \equiv \frac{\int_x Q_t^x \tilde{K}_{j,t+1}^x dx + \sum_i Q_t^{z_i} \tilde{K}_{j,t+1}^{z_i}}{N_{j,t+1}}. \quad (1.75)$$

The portfolio weight of each asset considered in the left side of equation (1.74) is given by

$$w_{j,t}^m \equiv \frac{Q_t^m \tilde{K}_{j,t+1}^m}{L_{j,t} N_{j,t+1}}, \quad (1.76)$$

for $m \in \{x, z_1, z_2, \dots, z_{N_z}\}$.

As in Section 1.4, the expected rate of return per unit of matched capital for the assets considered in the left-hand side of equation (1.74) is defined by

$$R_{j,t+1}^{k,z_i} \equiv \frac{r_{j,t+1}^{k,z_i} + (1 - \delta) [\bar{\psi} J_{t+1}^u + (1 - \bar{\psi}) Q_{t+1}^c]}{Q_t^{z_i}}, \quad (1.77)$$

for $i \in \{1, 2, \dots, N_z\}$, and

$$R_{j,t+1}^{k,x} \equiv \frac{\sum_i r_{j,t+1}^{k,z_i} f_z(z_i) + (1 - \delta) [\bar{\psi} J_{t+1}^u + (1 - \bar{\psi}) Q_{t+1}^c]}{Q_t^x}, \quad (1.78)$$

where similar to equation (1.43) in Section 1.4, net revenues from production per unit of effective capital matched at productivity z_i are defined by

$$r_{j,t}^{k,z_i} = \left(A_t z_i \left(\tilde{h}_{j,t}^{z_i} \right)^{1-\alpha} - W_t \tilde{h}_t^{z_i} \right) u_{j,t}^{z_i} - C_u(u_{j,t}^{z_i}). \quad (1.79)$$

The entrepreneurs' objective function (equation (1.49) in Section 1.4) can then be expressed as

$$E_t \left\{ \int_{\bar{\omega}_{t+1}}^{\infty} \left[\omega \tilde{R}_{j,t+1}^k L_{j,t} N_{j,t+1} - Z_{j,t+1} D_{j,t+1} \right] dF_{\omega}(\omega, \sigma_t) \right\}. \quad (1.80)$$

where, similar to Section 1.4, the portfolio return, denoted $\tilde{R}_{j,t+1}^k$, is defined by $R_{j,t+1}^k \equiv \int_x w_{j,t}^x R_{j,t+1}^{k,x} dx + \sum_i w_{j,t}^{z_i} R_{j,t+1}^{k,z_i}$.

Similarly, the financial intermediary's participation constraint (equation (1.53) in Section 1.4) can be expressed as

$$D_{j,t+1} R_t = [1 - F_{\omega}(\bar{\omega}_{t+1}, \sigma_t)] Z_{j,t+1} D_{j,t+1} + (1 - \mu_m) \int_0^{\omega_{t+1}} \omega dF_{\omega}(\omega, \sigma_t) \tilde{R}_{j,t+1}^{k,\psi} L_{j,t} N_{j,t+1}, \quad (1.81)$$

where $\tilde{R}_{j,t+1}^{k,\psi}$ denotes the portfolio return of *separated* capital, which, similar to Section 1.4, is defined by $\tilde{R}_{j,t+1}^{k,\psi} \equiv \int_x w_{j,t}^x R_{j,t+1}^{k,x,\psi} dx + \sum_i w_{j,t}^{z_i} R_{j,t+1}^{k,z_i,\psi}$.

From this, the entrepreneur's problem can proceed as in Section 1.4.

Equilibrium. As in Section 1.4, any submarket visited by a positive number of buyers must have the same price for capital in equilibrium, and buyers will be indifferent among them. Formally, for all x ,

$$\theta_t(x) \left(x + \frac{c_s}{q(\theta_t(x))} - Q_t^{\bar{z}} \right) = 0. \quad (1.82)$$

This condition determines the equilibrium market-tightness function: For all $x < Q_t$,

$$\theta_t(x) = q^{-1} \left(\frac{c_s}{Q_t^{\bar{z}} - x} \right). \quad (1.83)$$

For all $x \geq Q_t^{\bar{z}}$, $\theta_t(x) = 0$.

The mass of capital that transitions from employment to employment, denoted I_t^{ee} , is defined by

$$I_t^{ee} = \sum_{z_i} (1 - \psi_t) p(\theta_t(x_t^{z_i})) (1 - \delta) K_t^{z_i},$$

where $K_t^{z_i}$ denotes the stock of employed capital matched at productivity z_i in period t . This object will be the main focus of the next section, when studying the quantitative implications of this model for capital reallocation.

Similar to Section 1.4, market clearing in centralized markets for capital imply employed capital matched at productivity level z_i evolves then according to the law of motion,

$$K_{t+1}^{z_i} = K_t^{z_i} (1 - \psi_t) (1 - p(\theta_t(x_t^{z_i}))) + [I_t^{ue} + I_t^{ee}] f_z(z_i),$$

where I_t^{ue} denotes the mass of capital that transitions from unemployment to employment, that using the definition of market tightness, the law of large numbers, and the fact that a household's choice of submarket, $x_{i,t}$ is the same for all units of capital i , is given by $I_t^{ue} = p(\theta_t(x_t^u)) (1 - \delta) K_t^u$.

The remaining equilibrium conditions are similar to the model presented in Section 1.4.

1.6.2 Quantitative Analysis

This section studies some quantitative implications of the model regarding capital reallocation, using the estimated parameters values from Section 1.5. The only new functional form is that associated to the distribution of match-specific productivities. The discrete set of match-specific productivities is assumed to have three values, labeled low-, medium-, and high-match specific productivity. The steady-state distribution of match-specific productivities is assumed to be uniform. The dispersion between low- and high-match specific productivity is set to target a steady state

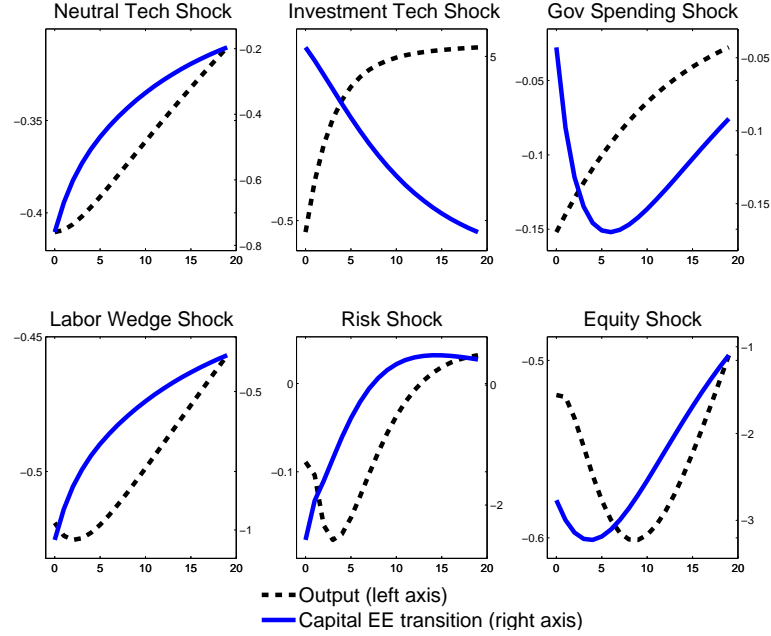


FIGURE 1.11: Impulse Responses to Contractionary Shocks.

Note: Response of capital employment-to-employment transitions and output to one-standard-deviation contractionary shocks. Labels *Neutral Tech Shock*, *Investment Tech Shock*, *Gov Spending Shock*, *Labor Wedge Shock*, *Risk Shock*, and *Equity Shock*, refer, respectively, to shocks to the variables A_t , A_t^I , G_t , φ_t , σ_t , and ζ_t presented in Section 1.4. Impulse responses expressed in percent deviations from steady state. Horizontal axes display quarters after the shock.

value of capital reallocation of 0.9% per quarter, the average of the range reported in Eisfeldt and Rampini (2006).

Procyclical capital reallocation. A well-documented stylized fact is that capital reallocation in the U.S. economy is procyclical (see Ramey and Shapiro, 1998; Eisfeldt and Rampini, 2006). The model presented in this section predicts a correlation between the mass of capital that transitions from employment to employment and output of 33.8%. in line with the range between 43.1% and 51.1% correlation between capital reallocation and output reported in Eisfeldt and Rampini (2006).

Figure 1.11 shows that, in response to most contractionary shocks, the mass of capital that transitions from employment to employment tends to fall, explaining the procyclical nature of capital reallocation. The explanation of this result through

TABLE 1.5
VARIANCE DECOMPOSITION

Shock		I^{ee}
Neutral technology	A	1.2
Investment-specific technology	A^I	49.8
Government spending	G	0.1
Labor wedge	φ	3.2
Risk	σ	5.8
Equity	ζ	39.9

the lens of the model is that contractionary shocks are generally associated with less demand of capital from entrepreneurs, which leads sellers visit submarkets with less favorable terms, both in terms of price of the units of capital and in terms of the probability of finding a buyer. Therefore, the same factors that lead to a countercyclical capital unemployment lead to a procyclical capital reallocation.

The estimated model can also be used to interpret the sources of fluctuations in capital reallocation. Table 1.5 shows the variance decomposition predicted by the model for the transition of capital from employment to employment and shows that most of the predicted capital-reallocation movements can be accounted by investment-specific productivity shocks (49.8%) and financial shocks (45.6%) . These findings are consistent those of Section 1.5, (most of the variation of capital unemployment can be explained by investment-specific shocks and financial shocks) and with those of previous literature explaining procyclical capital reallocation (Cui, 2013).

Misallocation. Empirical evidence points out that recession episodes, and in particular financial crises, are periods of misallocation (see, for example, Midrigan and Xu, 2014).

The predictions of the model presented in this section are also consistent with this empirical finding. Figure 1.12 shows the response to contractionary shocks of the mass of capital employed with a low match-specific productivity and the mass

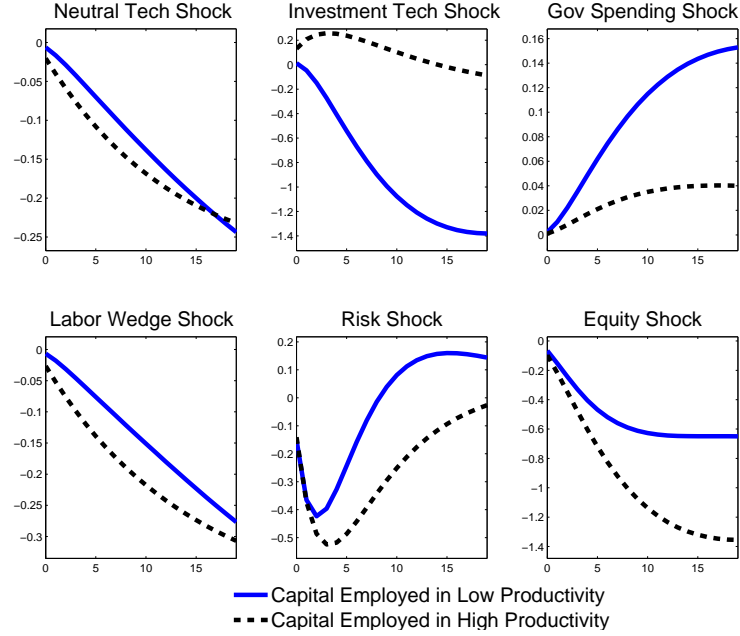


FIGURE 1.12: Impulse Responses to Contractionary Shocks.

Note: Response to one-standard-deviation contractionary shocks of the mass of capital employed with a low match-specific productivity ($K_t^{z_1}$, labeled *Low Productivity*) and the mass of capital employed at a high match-specific productivity ($K_t^{z_{Nz}}$ labeled *High Productivity*) predicted by the model presented in Section 1.6. Labels *Neutral Tech Shock*, *Investment Tech Shock*, *Gov Spending Shock*, *Labor Wedge Shock*, *Risk Shock*, and *Equity Shock*, refer, respectively, to shocks to the variables A_t , A_t^I , G_t , φ_t , σ_t , and ζ_t presented in Section 1.4. Impulse responses expressed in percent deviations from steady state. Horizontal axes display quarters after the shock.

of capital employed at a high match-specific productivity. The share of capital employed in match-specific productivity increases, especially in response to a negative equity shock (ζ_t). This is because reallocation is especially concentrated in units of capital employed at low match-specific productivity. Therefore, through the lens of this model, capital misallocation during crises is the other side of procyclical capital reallocation.

1.7 Conclusion and Future Research

This paper presented a model with investment search frictions in which financial shocks have a sizable effect in macroeconomic variables though capital unemploy-

ment. An estimated version of the model for the U.S. economy shows that the proposed mechanism can lead to investment slumps such as the one observed during the Great Recession. This result is relevant because slow investment recoveries typically characterize financial crisis episodes.

Using the estimated version of the model to interpret the sources of business-cycle fluctuations in the U.S. economy, the model assigns a large role (33% of output fluctuations) to financial shocks, in the context of a real model that would have assigned a negligible role to these shocks (1% of output fluctuations). This result is relevant because an important source of discrepancy between real and monetary business-cycle models is the role assigned to financial shocks. This paper shows that incorporating investment search frictions can reconcile an important part of this discrepancy. Finally, the paper shows that the framework can be used to explain capital reallocation and misallocation during crises, as documented by previous empirical literature.

The findings of this paper suggest that two related areas of future research could be promising to develop. The first area is normative. As shown in the paper, the directed-search framework studied leads to an efficient allocation. However, combining the search frictions considered in this paper with asymmetric information would lead to a scope for policy related to asset purchases and subsidy programs as shown in [Guerrieri and Shimer \(2014\)](#).

The second area for future research is empirical. In particular, future research could explore more direct evidence of investment search frictions. For instance, it would be possible to investigate the existence of a “Beveridge curve” in the physical-capital market, using data from capital-intermediary firms. It would also be possible to study the testable implications developed from the model in this paper regarding the relationship between capital unemployment, economic activity and investment. This could be done, for instance, using geographical data of the sort used in this paper to measure capital unemployment. These extensions are planned for future

research.

Labor Market, Financial Crises and Inflation: Jobless and Wageless Recoveries

Guillermo Calvo, Fabrizio Coricelli, and Pablo Ottonello

2.1 Introduction

The slow recovery of unemployment has been one of the most salient features in the policy debate that accompanied the Great Recession. The fact that, in the context of high unemployment, US output has recovered its precrisis level has lead many analysts, in both academic and policy circles, to label the pattern a “jobless recovery” (see Figure 2.1). In Europe, the pattern of unemployment recovery seems to be even more dramatic: Six years after the recession began, unemployment has not yet begun to recover its precrisis level.

This paper casts light on the reasons for jobless recovery in the Great Recession by studying labor-market recovery in a sample of 116 postwar recession episodes—prior to the Great Recession—in developed (DMs) and emerging market economies (EMs). We document two new stylized facts. First, in “low-inflation” recession episodes (i.e., annual inflation below 30 percent), financial crises tend to be followed by greater unemployment than in other recession episodes. Second, in “high-inflation” recession episodes, financial crises are not followed by jobless recoveries but by “wageless recoveries,” characterized by a lower real wage once output recovers its trend. These findings are summarized in Figure 2.2, which compares the behavior of DMs’ and

EMs' respective labor markets during financial crises, relative to other episodes.

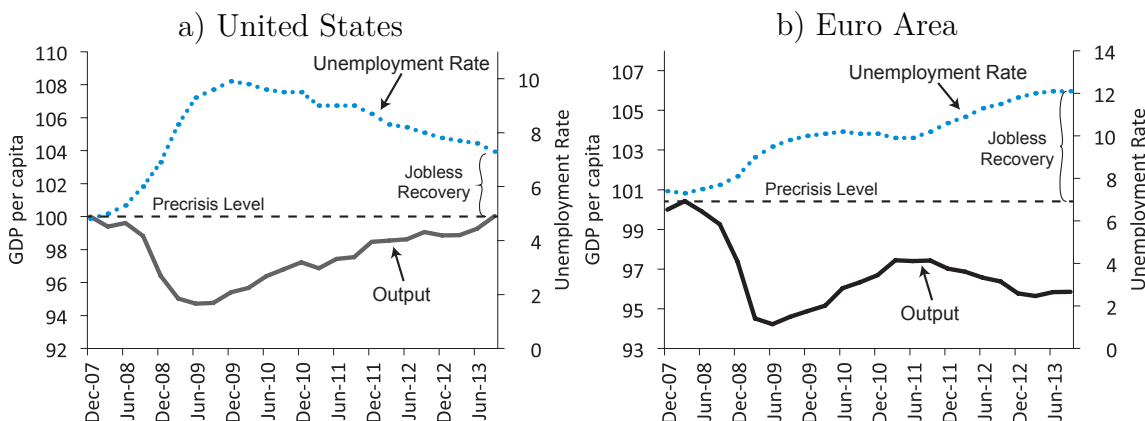


FIGURE 2.1: Jobless recovery during the Great Recession

Notes: Euro Area includes EA-17, Eurostat definition; GDP in real terms, *peak* = 100; unemployment rate in percent. Seasonally adjusted figures.

In DMs, where inflation in the postwar era has been relatively low, financial crises have been followed by recoveries in which joblessness was significantly higher than in other recessions. This is in line with [Reinhart and Reinhart \(2010\)](#): During the ten years following financial crises, unemployment rates remain on average five percentage points above the average rate ten years prior to the crisis. Similar evidence is provided by [Knotek and Terry \(2009\)](#), who show that, for the “big five” banking crises (Spain 1977, Norway 1987, Finland 1991, Sweden 1991, Japan 1992), unemployment rates have been higher and more persistent than in recessions not associated with banking crises.

In EMs, there is a much higher dispersion in inflation rates during financial crises. Exploiting these differences in inflation rates, we find again a sluggish adjustment of labor markets during the recovery from financial crises, but the nature of such adjustment varies with inflation. High-inflation recession episodes are not associated with jobless recoveries but with wageless recoveries. In contrast, low-inflation EMs display a pattern similar to that observed in DMs, with financial crises associated with more intense jobless recoveries. The findings are in line with models of nominal wage rigidities, where a price spike would lower the rate of unemployment (for a

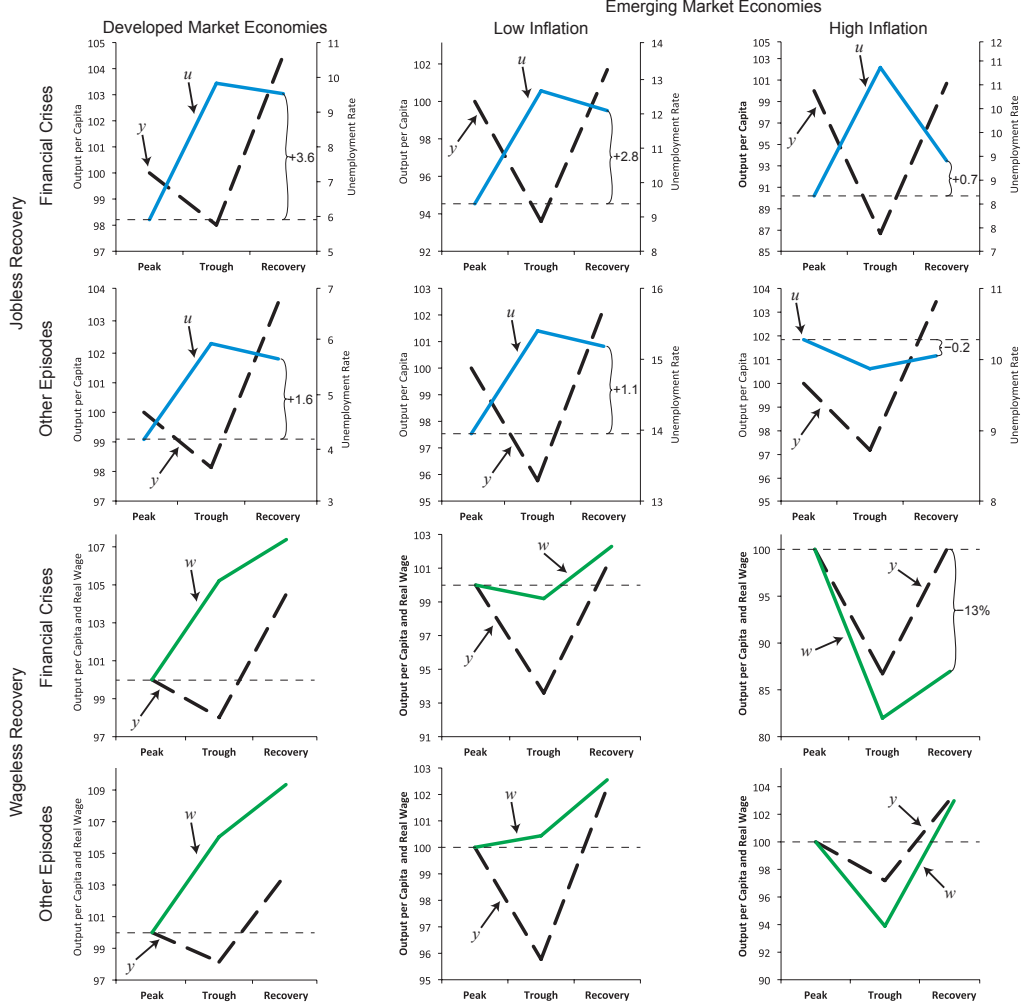


FIGURE 2.2: Labor Market Recovery, Financial Crises and Inflation

Notes: See Section 2.2 for a description of the sample and data; u refers to the unemployment rate, in percent; y refers to real GDP per capita, and w refers to real wages, $peak = 100$.

recent study in this direction showing the importance of wage rigidity in a crisis environment, see [Schmitt-Grohe and Uribe, 2013](#)).

The paper conducts an econometric analysis finding that the association between financial crises and jobless and wageless recoveries, as shown in Figure 2.2, is robust to controlling for countries' characteristics (such as labor-market indicators, secular growth, financial development, and country size) and to characteristics of the recession episodes (such as duration of the episode or the depth of the output contraction). To provide evidence on the effect of financial crises in this association, we also carry out an instrumental variable (IV) strategy using credit-market outcomes prior to the

recession episode in order to identify the exogenous effect of financial crises on jobless recoveries.

A common explanation given for the high unemployment observed in the Great Recession is that output has not recovered its trend. In this line, several papers have recently argued that jobless recoveries are not a pattern observed in the data, based on the stability of Okun’s law (e.g., [Ball, Leigh and Loungani, 2013](#); [Galí, Smets and Wouters, 2012](#)). We show that a key difference between our findings and the results obtained in this literature is related to the measure of jobless recovery used. Estimations of Okun’s law typically focus on the cyclical component of unemployment. In our study, we measure jobless recovery as the change in unemployment rate from output peak to recovery. In fact, if we were to measure jobless recoveries as deviations from the “natural rate,” we would also find little trace of jobless recovery. Our evidence suggests that jobless recoveries mostly occur at lower frequencies than the ones typically studied in Okun’s law regressions. At first sight, this result could be interpreted as a sign that financial crises are related to changes in the natural rate of unemployment (for a theoretical formulation and evidence related to this hypothesis, see [Acemoglu, 2001](#); [Dromel, Kolakez and Lehmann, 2009](#)). However, our evidence that high-inflation recession episodes do not display jobless recovery suggests that this might be better characterized as persistent unemployment in the presence of low inflation and nominal rigidities—and therefore that a policy of generating a spike in inflation might succeed in reducing the unemployment rate.

To rationalize the findings of our empirical study, we develop a simple analytical framework in which financial crises—formalized as an exogenous contraction of collateral constraints—can lead to jobless recoveries. The key assumption is that collateral requirements are lower for projects and firms possessing easily recognizable collateral, such as physical capital. We thus show that, as a result of a collateral crunch, firms choose to employ more capital-intensive techniques, implying jobless recovery under

wage rigidity. It is worth noting that, despite the simplicity of the model, collateral and other financial issues have not played a central role in the theoretical literature concerned with jobless recoveries (see [Schreft, Singh and Hodgson, 2005](#); [Shimer, 2012](#); [Berger, 2012](#); [Jaimovich and Siu, 2012](#); [Schmitt-Grohé and Uribe, 2012b](#)).¹ We then test the role of collateral for the sample of DMs, using data on asset prices (house prices) as a proxy for collateral values, and we find that, in a low-inflation context, the recovery of collateral variables is significantly associated with jobless recoveries.

The rest of the paper is organized as follows. Section [2.2](#) describes the sample of recession episodes and the variables used in the empirical analysis. Section [2.3](#) documents the association between financial crises and jobless and wageless recoveries and provides evidence from an instrumental variables strategy. Section [2.4](#) studies the results' robustness to the inclusion of additional controls and to the use of other measures of financial crises and jobless recovery. Section [2.5](#) presents an analytical framework to rationalize the association between financial crises and jobless recovery. Section [2.6](#) concludes.

2.2 Data and Descriptive Statistics

2.2.1 Sample Construction

2.2.1.1 Developed- and Emerging-Market Recession Episodes

To analyze the relationship between financial crises and labor-market recovery, we construct two samples of recession episodes: one for DMs and one for EMs. Constructing two separate samples allows us to use quarterly data in the DMs.

¹ Financial considerations *do* play a key role in the dynamics of employment in both the theoretical and the empirical literature (for a recent survey see [Brunnermeier, Eisenbach and Sannikov, 2012](#)). However, the phenomenon of jobless recovery is different from that of employment fluctuations; it implies delinking employment from output.

Using quarterly data, we construct a sample of recession episodes during the period 1950–2006 for 11 DMs: Austria, Australia, Canada, France, Germany, Italy, Spain, Sweden, Switzerland, the United Kingdom and the United States. We use the NBER (for the US) and the ECRI (for the other economies) recession dates to identify the occurrence of a recession event.²

For EMs, due to limited data availability, we use annual data and construct a sample of recession episodes from 1980 to 2006. Following [Calvo, Izquierdo and Talvi \(2006\)](#), we identify the occurrence of a recession event as a period of negative annual change in GDP. To reduce heterogeneity among EMs, we focus on countries that are integrated into the world capital market, defined as countries included in JP Morgan’s Emerging Market Bond Index (EMBI). Countries included in the sample are Argentina, Brazil, Bulgaria, Chile, Colombia, Croatia, the Czech Republic, the Dominican Republic, Ecuador, El Salvador, Hungary, Indonesia, Ivory Coast, Lebanon, Malaysia, Mexico, Morocco, Nigeria, Panama, Peru, the Philippines, Poland, Russia, South Africa, South Korea, Thailand, Tunisia, Turkey, Ukraine, Uruguay, and Venezuela.³

For each recession episode in a DM or EM, we define an output peak, trough, and recovery point using the cyclical component of output per capita. In particular, given a recession episode, we define an output peak as the period displaying the maximum cyclical component of output per capita in the window with a positive cyclical component of output per capita preceding the recession event.⁴ The recovery

²NBER and ECRI follow similar methodologies to define and date recessions. Countries were selected on the basis of data and recession dates’ availability. Japan was not considered due to its strong idiosyncratic differences during this period. We did not include in the sample the 1995 episode in Austria, defined by the ECRI as recession, because there was no output contraction.

³Since we are interested in analyzing the recovery of unemployment in market economies during the crisis, we excluded from this sample episodes associated with the dissolution of the Soviet Union (in particular, the recession episodes that started prior to 1991 in Bulgaria, the Czech Republic, Croatia, Hungary, Poland, Russia and Ukraine).

⁴If no observation with a positive cyclical component of output exists between the trough of a

point is defined as the period, after a recession event, in which output per capita recovers its trend level. The output trough is defined as the period between output peak and recovery point displaying the minimum level of the cyclical component of output per capita. Since we are studying the pattern of the recovery from recession episodes, we do not include in the sample episodes in which output per capita did not fully recover its trend before the occurrence of another recession episode. We compute the cyclical component of output using a Hodrick–Prescott (HP) filter with a smoothing parameter of 1600 for quarterly data and 100 for annual data ([Hodrick and Prescott, 1997](#); [Ravn and Uhlig, 2002](#)). Defining the recovery point of output per capita in terms of its trend level ensures that differences among episodes are not driven by different recoveries to trend. Results do not significantly change if we define the recovery point as the point in which output recovers its precrisis level rather than its trend. Data on output and population are obtained from OECD, WEO, and WDI datasets.

With this methodology, we obtain a sample of 45 DM recession episodes, and 71 EM recession episodes, listed in Table [B.1](#) of Appendix [B.1](#). Next we classify recession episodes according to the inflation rate exhibited during the recession episode.

2.2.1.2 Low- and High-Inflation Recession Episodes

A major difference between DMs and EMs is that recession episodes in the latter tend to display much higher inflation, as shown in Figure [2.3](#). In the presence of nominal wage rigidities, inflation is a potential mechanism to induce a contraction of real wages and thus restore full employment (see, for example, [Galí, 2011](#), [Schmitt-Grohe and Uribe, 2013](#)). To explore this hypothesis, we divide the sample of EMs into “low inflation” episodes and “high inflation” episodes. For each episode, we compute

previous recession episode and beginning of the recession event, the output peak is simply defined as the period displaying the maximum cyclical component of output per capita between the trough of the previous recession episode and the beginning of the recession event.

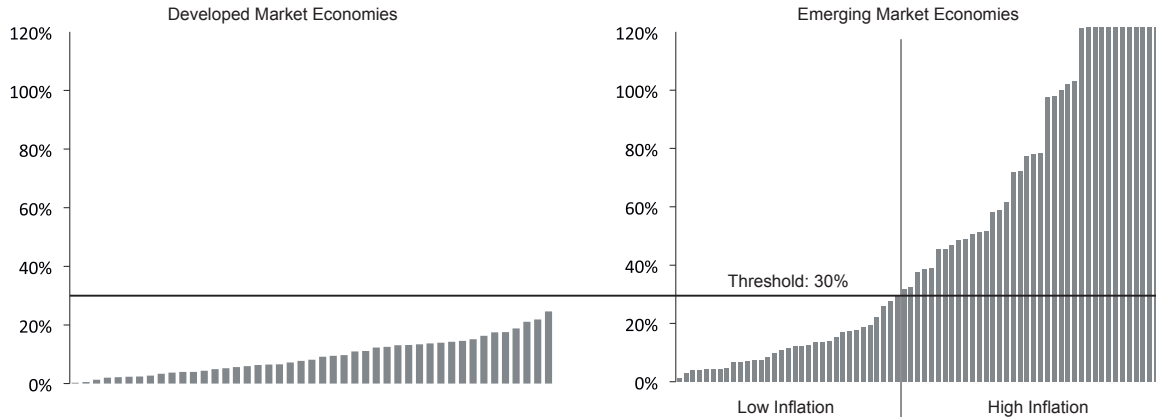


FIGURE 2.3: Inflation in Recession Episodes

Notes: Inflation refers to maximum level of annual inflation observed during the episode; See Section 2.2.1 for a description of the sample and data.

Data Source: IMF

the maximum level of inflation for the entire episode. We compute inflation using the producer price index (wholesale price index or the consumer price index when not available) obtained from the IMF dataset and national sources. The maximum annual level of inflation observed in a DM recession episode is 24.6 percent. We define a high- (low-) inflation episode as one in which the maximum level of the annual rate of inflation is above (below) 30 percent.⁵ The threshold considered is the upper bound identified in [Dornbusch and Fischer \(1993\)](#) to define moderate inflation, and the cutoff above which ? define high inflation. With this threshold, low-inflation EMs have an average inflation of 11.9 percent, not statistically different from the average DM inflation (9.4 percent). The standard deviation is also similar: 7.4 percent for low-inflation EMs and 6.2 percent for DMs. Thus, the distribution of low-inflation EMs is comparable, in terms of inflation during recession episodes, to that of DMs.

⁵In [Calvo, Coricelli and Ottonello \(2014\)](#), we conduct a threshold estimation, following [Hansen \(2000\)](#), to identify a level of inflation from which EM financial-crisis episodes have a different degree of jobless recovery. Results confirm the presence of a threshold around 30 percent (point estimate of 31.7 percent). We also study whether, in EM financial crises, one can establish a linear relationship between the inflation experienced in the episode (the level of inflation or the change in inflation) and unemployment recovery. We uncovered no strong evidence supporting the statistical significance of a linear relationship between a continuous measure of inflation and unemployment recovery.

2.2.2 Definition of Variables

In this section we describe the data sources and the construction of the variables used in the empirical analysis.

2.2.2.1 Measures of Jobless and Wageless Recovery

To measure jobless recovery, we compute, for each episode, the change in the unemployment rate between output peak and output recovery point ($\Delta_{PR}u$). Looking at the *change* in the unemployment rate permits us to abstract from historical differences in the average unemployment rate in these economies, which is likely to be determined by structural characteristics and labor-market institutions. In the Section 2.4, we study the robustness of the results to alternative measures of jobless recovery. Similarly, to measure wageless recovery, we computed, for each episode, the change in the (log) real wage between output peak and output recovery point ($\Delta_{PR}w$). The data on unemployment and wages were obtained from WEO, ILO and ECLA datasets and from national sources. Nominal wages were deflated by the wholesale price index or producer price index, obtained from OECD and IFS datasets and national sources.

2.2.2.2 Financial-Crisis Episodes

For each recession episode, we construct a dummy variable (*fin_crisis*) that takes the value of one if a banking crisis event or a debt default/rescheduling event occurs in a window from 1 year before the output per capita peak to 1 year after the output per capita recovery point. This classifies 13 DM episodes as financial crises (29 percent of the sample) and 57 EM episodes (80 percent of the sample), detailed in Table B.1 of Appendix B.1. Data on banking crises and debt default/rescheduling events are obtained from Reinhart and Rogoff (2009a).

2.2.2.3 Control Variables

The baseline empirical analysis includes two sets of controls (the set of controls is further expanded in Section 2.4). First, we control for labor-market indicators (denoted by *labor_mkt_P*) computed at the output peak. As emphasized in the labor-market literature, labor-market institutions are likely to affect the response of unemployment to shocks, including the recovery of unemployment following recession episodes (see, for example, Bertola, Blau and Kahn, 2002; Blanchard, 2006; Furceri and Mourougane, 2009; Bernal-Verdugo, Furceri and Guillaume, 2012). We use two variables: a *de jure* indicator of labor-market legislation (*lamrig_P*) from the recent dataset on labor-market regulations constructed by Campos and Nugent (2012);⁶ and a *de facto* measure of labor-market rigidities, namely the natural rate of unemployment (*natural_u_P*), which is likely to be affected by labor-market institutions. For DMs, we use the natural rate of unemployment reported in the IMF–WEO dataset. For EMs we compute the average rate of unemployment in the whole sample period as a proxy for the natural rate of unemployment (to the best of our knowledge, there is no dataset available that reports the natural rate of unemployment for a large set of EM countries).

Second, we control for the secular growth experienced throughout the recession episode, denoted by *gd*. With *g* denoting the annual secular growth rate of a given country and *d* the duration of a recession episode, the secular growth experienced throughout the recession episode is defined as $gd = g \times d$. The secular growth rate for a given country is computed as the average per capita growth rate for the sample period. The duration of the recession episode is defined as the number of years from output peak to recovery point. Controlling for this variable is relevant since

⁶The variable *lamrig* is an index of labor-market legislation rigidity, constructed in Campos and Nugent (2012) by reviewing labor-market legislation. This new index extends both in terms of country coverage and of time span the widely used OECD dataset on employment protection legislation (see also Botero et al., 2004).

TABLE 2.1
DESCRIPTIVE STATISTICS

Variable:	Developed			Emerging Market Economies								
	Market Economies			All Episodes			Low Inflation			High Inflation		
	All	Financial	Other	All	Financial	Other	All	Financial	Other	All	Financial	Other
	Episodes	Crises	Episodes	Episodes	Crises	Episodes	Episodes	Crises	Episodes	Episodes	Crises	Episodes
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$\Delta_{PR}u$	0.022 (0.003)	0.036 (0.007)	0.016 (0.002)	0.013 (0.004)	0.015 (0.004)	0.005 (0.006)	0.023 (0.005)	0.028 (0.006)	0.011 (0.006)	0.006 (0.005)	0.007 (0.005)	−0.002 (0.011)
$\Delta_{PR}w$	0.084 (0.022)	0.071 (0.033)	0.089 (0.029)	−0.052 (0.030)	−0.074 (0.037)	0.027 (0.026)	0.023 (0.037)	0.023 (0.051)	0.025 (0.038)	−0.114 (0.043)	−0.140 (0.049)	0.029 (0.034)
$natural_u_P$	0.057 (0.006)	0.069 (0.009)	0.052 (0.007)	0.106 (0.007)	0.101 (0.007)	0.124 (0.016)	0.119 (0.013)	0.112 (0.017)	0.138 (0.019)	0.097 (0.007)	0.096 (0.007)	0.107 (0.025)
$lamrig_P$	1.282 (0.144)	1.406 (0.321)	1.230 (0.156)	1.660 (0.057)	1.670 (0.067)	1.621 (0.097)	1.537 (0.082)	1.505 (0.101)	1.639 (0.130)	1.767 (0.075)	1.798 (0.083)	1.597 (0.160)
gd	0.082 (0.006)	0.095 (0.010)	0.077 (0.007)	0.083 (0.015)	0.084 (0.017)	0.078 (0.036)	0.066 (0.019)	0.063 (0.023)	0.076 (0.033)	0.083 (0.015)	0.084 (0.017)	0.078 (0.036)

Notes: Standard errors in parentheses. Sample and variables definition are detailed in Section 2.2.

countries can have different long-run growth rates and recession episodes might differ in their duration, which can affect jobless and wageless recoveries. For instance, in a standard growth model, higher technological progress would lead to a higher growth of real wages.

2.2.3 Descriptive Statistics

Table 2.1 presents summary statistics for the sample of recession episodes, splitting the sample into DM and EM episodes and the latter into low- and high-inflation episodes. Columns 1–3 indicate that the average DM recession episode displays a statistically significant jobless recovery (from output peak to recovery, unemployment increases 2.2 percentage points) but no negative effect on wages (from output peak to recovery, wages increase 8.4 percent). If we split the DM sample between financial crises and other recession episodes, we see that financial crises display a greater increase in unemployment: 3.6 percent in financial crises vs. 1.6 percent in other recession episodes (see also Figure 2.2).

Columns 4–6 indicate that the average EM recession episode displays both statistically significant jobless and wageless recovery, driven by financial-crisis episodes (other recession episodes do not display a statistically significant jobless or wageless recovery). However, Columns 7–12 show that splitting the sample in low and high inflation uncovers two very different patterns: low-inflation financial-crisis episodes display a statistically significant jobless recovery (from output peak to recovery, unemployment increases 2.8 percentage points) and no wageless recovery, while high-inflation financial-crisis episodes display no statistically significant jobless recovery and a large and statistically significant wageless recovery (from output peak to recovery, real wages contract 13 percent). As an illustration of this pattern see Figure 2.2.

Table 2.1 also shows the descriptive statistics of the baseline controls, which indicate that financial crises tend to occur more often in a context of higher labor-market rigidities, and to have a larger duration (which is reflected in the control variable *gd*). DM financial crises tend to occur more often in the context of a high natural rate of unemployment, whereas EM financial crises tend to occur in the context of a low natural rate of unemployment. These differences in the raw data point to the relevance of controlling for different characteristics of the recession episodes and of labor markets to identify the association between financial crises and labor-market recovery (Section 2.4 expands further this set of controls).

2.3 Econometric Analysis

2.3.1 Methodology

The baseline empirical model relates jobless and wageless recoveries to financial crises, controlling for labor-market characteristics and secular growth:

$$\Delta_{PR}z_i = \alpha + \beta fin_crisis_i + X_i'\gamma + \epsilon_i. \quad (2.1)$$

where $\Delta_{\text{PR}}z_i$ denotes the jobless recovery measure ($\Delta_{\text{PR}}u_i$) or wageless recovery measure ($\Delta_{\text{PR}}w_i$) in recession episode i , \mathbf{X} is a vector of controls including labor-market controls ($labor_mkt_{\text{P},i}$) and secular growth (gd_i), and ϵ_i is a random error term (variables are defined in Section 2.2). The coefficient of interest is β , the difference in jobless recovery or wageless recovery displayed by financial-crisis episodes relative to other episodes.

The ordinary least squares (OLS) estimates of Equation (2.1) provide evidence for the association between financial crises and jobless recovery, but they cannot suggest any causality: Financial crises can be endogenous to jobless recoveries. For example, an increase in the unemployment rate driven by technological factors could induce a fall in house prices and a decrease in collateral values, triggering a financial crisis. We provide some evidence on the effect of financial crises on jobless and wageless recoveries using an IV strategy. The instrument is a variable that captures credit-market outcomes *prior* to the recession episode, as is typically done in the literature to predict financial crises (see, for example, [Mendoza and Terrones, 2012](#); [Schularick and Taylor, 2012](#); [Gourinchas, Valdes and Landerretche, 2001](#)). Specifically, we use the cyclical component of real per capita credit at the output peak ($credit_{\text{P}}$).⁷ Data on credit were obtained from the IFS dataset and from national sources.

Table 2.2 shows the first-stage relationship for DMs and EMs. The first-stage coefficients are statistically significant at the one- and 10-percent levels, showing that credit booms prior to recession episodes are associated with a higher probability of the recession being financial.

⁷The cyclical component of real per capita credit was obtained using an HP filter. Results do not change when we use a log quadratic trend to compute the cyclical component.

TABLE 2.2

FIRST STAGE: CREDIT CYCLE AT THE OUTPUT PEAK AND FINANCIAL CRISES

Dependent Variable:	<i>fin_crisis</i>								
	Developed			Emerging Market Economies					
	Market Economies			Low Inflation			High Inflation		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
<i>credit_P</i>	4.448*** (1.515)	4.900*** (1.488)	4.747*** (1.514)	0.737* (0.386)	0.744* (0.419)	0.768* (0.403)	0.997*** (0.235)	0.970*** (0.251)	1.008*** (0.241)
<i>natural_u_P</i>		2.474 (2.137)			−0.386 (1.887)			0.108 (0.602)	0.448 (0.618)
<i>lamrig_P</i>			−0.015 (0.079)			−0.023 (0.170)			0.242* (0.134)
<i>gd</i>		1.399 (1.972)	2.818 (1.906)		0.470 (1.133)	0.619 (0.885)		−1.476 (1.554)	
Observations	45	45	44	23	23	23	33	33	33

Notes: Standard errors in parentheses. * indicates significance at 10 percent level; ** at 5 percent level; *** at 1 percent level. Sample and variables definition are detailed in Section 2.2.

2.3.2 Empirical Results

Estimation results of Equation (2.1), linking financial crises to jobless and wageless recoveries, are reported in Table 2.3. Results for DMs are reported in Panel A. Columns 1–4 show the association between jobless recoveries and financial crises. The OLS estimates, reported in Columns 1 and 2, indicate that there is a positive and statistically significant association between financial crises and jobless recoveries. Columns 3 and 4 show that the IV estimates are also positive and significant, providing evidence that the exogenous component of financial crises helps explain jobless recoveries. Note that the IV coefficients are larger than those of the OLS model, suggesting that the endogeneity of unemployment and financial crises could underestimate the effects. The magnitude of the coefficients suggests that jobless recoveries can be significantly larger during financial crises: When output per capita recovers its precrisis trend, the divergence from the unemployment rate at its precrisis level tends to be between 1.8 and 2.8 percentage points greater than in a regular recession. Note that these figures are similar to those observed in the United States and Europe during the global financial crisis that started in 2008 (see Figure 2.1). Columns

5–8 show the association between wageless recoveries and financial crises. None of the coefficients of the OLS or IV regressions is statistically significant. Therefore, in DMs, evidence suggests that financial crises are associated with jobless recoveries but not with the dynamics of real wages. In particular, there is no trace of wageless recoveries.

The results for low-inflation EMs are reported in Table 2.3, Panel B. As for DMs, evidence from OLS and IV estimates suggests that financial crises are associated with jobless recoveries (Columns 1–4) but not with wageless recoveries (Columns 5–8). The magnitude of the coefficient of jobless recoveries are similar to the one found for DMs.

The results for high-inflation EMs are reported in Table 2.3, Panel C. In sharp contrast with DMs and low-inflation EMs, financial crises in high-inflation EMs experience wageless rather than jobless recoveries. In Columns 1–4, both the OLS and IV estimates show that financial crises have no statistically significant association with unemployment recovery. On the other hand, the association between financial crises and the recovery of real wages is negative and statistically significant, as shown by the OLS estimates in Columns 5 and 6. Moreover, Columns 7 and 8 show that the IV estimates are also statistically significant, providing some evidence that the exogenous component of financial crises can be important in wageless recoveries.

2.4 Robustness

In this section, we investigate the robustness of the results reported in Section 2.3. In particular, we explore the robustness of the conclusions when i) we use an alternative measure of financial crises, ii) we include additional controls, and iii) we use alternative measures of jobless recovery.

TABLE 2.3
FINANCIAL CRISES AND LABOR MARKET RECOVERY

Dependent Variable:	Δ_{PRu}				Δ_{PRw}			
	OLS (1)	OLS (2)	IV (3)	IV (4)	OLS (5)	OLS (6)	IV (7)	IV (8)
Panel A: Developed Market Economies								
<i>fin_crisis</i>	0.018*** (0.005)	0.019*** (0.005)	0.028** (0.011)	0.027** (0.011)	-0.032 (0.052)	-0.040 (0.048)	-0.141 (0.164)	-0.147 (0.159)
<i>natural_up</i>	-0.003 (0.074)		-0.018 (0.079)		-0.138 (0.751)		0.173 (0.913)	
<i>lamrig_P</i>		0.003 (0.003)		0.003 (0.003)		-0.064** (0.026)		-0.068** (0.028)
<i>gd</i>	0.136* (0.069)	0.112* (0.065)	0.121 (0.074)	0.092 (0.071)	0.857 (0.684)	1.602** (0.603)	0.925 (0.736)	1.873** (0.755)
Observations	45	44	45	44	35	34	35	34
Panel B: Emerging Market Economies – Low Inflation								
<i>fin_crisis</i>	0.020* (0.010)	0.018* (0.010)	0.076* (0.043)	0.061* (0.035)	0.039 (0.107)	-0.009 (0.086)	-0.401 (0.422)	-0.308 (0.299)
<i>natural_up</i>	0.121 (0.084)		0.196 (0.149)		1.221 (0.986)		0.118 (1.711)	
<i>lamrig_P</i>		0.001 (0.008)		0.002 (0.011)		-0.019 (0.082)		-0.039 (0.102)
<i>gd</i>	-0.019 (0.050)	-0.062 (0.043)	-0.016 (0.083)	-0.080 (0.061)	0.480 (0.525)	0.202 (0.395)	0.406 (0.746)	0.342 (0.505)
Observations	23	23	23	23	21	27	21	27
Panel C: Emerging Market Economies – High Inflation								
<i>fin_crisis</i>	0.011 (0.014)	0.011 (0.014)	0.024 (0.025)	0.024 (0.023)	-0.247** (0.114)	-0.236* (0.118)	-0.397* (0.202)	-0.359* (0.196)
<i>natural_up</i>	0.043 (0.126)		0.056 (0.130)		-1.644 (1.193)		-1.951 (1.273)	
<i>lamrig_P</i>	-0.023 (0.054)	-0.022 (0.058)	-0.031 (0.056)	-0.034 (0.061)	1.113** (0.444)	1.181** (0.498)	1.232** (0.475)	1.317** (0.535)
<i>gd</i>		0.000 (0.012)		-0.002 (0.013)		0.080 (0.110)		0.107 (0.118)
Observations	35	35	35	35	33	33	33	33

Notes: Standard errors in parentheses. * indicates significance at 10 percent level; ** at 5 percent level; *** at 1 percent level. Sample and variables definition are detailed in Section 2.2.

2.4.1 Alternative Measure of Financial Crises

The measure of financial crises used in the baseline specification is a dummy variable based on the occurrence of banking crises or default/rescheduling events during the recession window. In this section, we study the robustness of the findings to the use of a continuous measure of financial crises and credit-market conditions: the contrac-

tion in credit during the recession episode. In particular, the alternative measure of financial crises is defined as the change in the cyclical component of real credit per capita from output peak to recovery point ($\Delta_{PR}credit_c$).⁸

We estimate the model defined in Equation (2.1) with the alternative measure of financial crises:

$$\Delta_{PR}z_i = \alpha + \beta\Delta_{PR}credit_c_i + X_i'\gamma + \epsilon_i. \quad (2.2)$$

Table 2.4 indicates that the results using the alternative measure of financial crises are similar to those obtained in the baseline specification. In particular, Panel A shows that in DMs, creditless recoveries are associated with jobless recoveries and seem unrelated to the recovery of real wages. Panel B shows that the same pattern is observed in low-inflation EMs. Finally, Panel C reports that in high-inflation EMs creditless recoveries are associated with wageless recoveries and not jobless recoveries. In summary, focusing on continuous indicators of credit conditions, rather than dummy variables identifying financial crises, broadly confirms the results obtained in the financial-crisis analyses.

2.4.2 Additional Controls

In this section, we study the robustness of our results to the inclusion of additional controls that could be associated with jobless recoveries and financial crises. A first source of concern could be that the association between financial crises and labor-market recovery is driven by characteristics of financial crises relative to other recession episodes that are unrelated to financial factors. For instance, financial crises are typically associated with a larger output contraction than other recession episodes

⁸In the recession episodes in which a financial crisis occurs prior to or at the output peak, we consider the maximum level in the cyclical component of real per capita credit between the beginning of the financial crisis and the output peak instead of the cyclical component of real per capita credit at the output peak. Indeed, when a financial crisis starts before the recession episode, the level of credit at the output peak is already affected by the financial-crisis episode. The cyclical component of credit was computed using the HP filter, but results do not change if we use a log quadratic trend.

TABLE 2.4
CREDIT RECOVERY AND LABOR MARKET RECOVERY

Dependent Variable:	Δ_{PRu}				Δ_{PRw}			
	OLS (1)	OLS (2)	IV (3)	IV (4)	OLS (5)	OLS (6)	IV (7)	IV (8)
Panel A: Developed Market Economies								
$\Delta_{PR} credit$	-0.104** (0.040)	-0.104** (0.040)	-0.109** (0.045)	-0.103** (0.044)	0.654 (0.517)	0.847* (0.474)	0.600 (0.640)	0.606 (0.586)
$natural_u_P$	0.041 (0.080)		0.042 (0.080)		-0.230 (0.721)		-0.230 (0.722)	
$lamrig_P$		0.003 (0.003)		0.003 (0.003)		-0.069** (0.025)		-0.067** (0.025)
gd	0.152** (0.074)	0.146** (0.070)	0.151** (0.074)	0.147** (0.070)	0.862 (0.670)	1.616*** (0.572)	0.860 (0.671)	1.584** (0.576)
Observations	45	44	45	44	35	34	35	34
Panel B: Emerging Market Economies – Low Inflation								
$\Delta_{PR} credit$	-0.034** (0.014)	-0.032** (0.015)	-0.048*** (0.016)	-0.041** (0.017)	0.470 (0.281)	0.440* (0.233)	0.472 (0.326)	0.367 (0.269)
$natural_u_P$	0.119 (0.081)		0.129 (0.083)		0.340 (0.999)		0.337 (1.037)	
$lamrig_P$		0.001 (0.008)		0.002 (0.008)		0.014 (0.078)		0.009 (0.079)
gd	-0.014 (0.048)	-0.057 (0.041)	-0.011 (0.050)	-0.058 (0.041)	0.313 (0.497)	0.348 (0.374)	0.312 (0.501)	0.323 (0.378)
Observations	23	23	23	23	21	27	21	27
Panel C: Emerging Market Economies – High Inflation								
$\Delta_{PR} credit$	-0.002 (0.016)	-0.004 (0.017)	-0.020 (0.021)	-0.022 (0.022)	0.277* (0.147)	0.290* (0.153)	0.380* (0.193)	0.367* (0.198)
$natural_u_P$	0.028 (0.134)		-0.019 (0.140)		-1.009 (1.192)		-0.962 (1.204)	
$lamrig_P$		0.003 (0.013)		0.009 (0.014)		-0.020 (0.111)		-0.033 (0.113)
gd	-0.017 (0.054)	-0.012 (0.058)	-0.022 (0.055)	-0.008 (0.059)	1.038** (0.447)	0.969* (0.484)	1.083** (0.454)	0.982* (0.487)
Observations	35	35	35	35	33	33	33	33

Notes: Standard errors in parentheses. * indicates significance at 10 percent level; ** at 5 percent level; *** at 1 percent level. Sample and variables definition are detailed in Section 2.2.

(see [Reinhart and Rogoff, 2009b](#)). Jobless recoveries could result from deeper recession episodes if, for example, larger output contractions lead to greater increases in unemployment and there is hysteresis in unemployment.⁹ A second source of concern is that country characteristics, such as labor-market or financial-development indicators, are associated simultaneously with a higher frequency of financial crises and with jobless recoveries. The two sets of controls we have included in the baseline specification are aimed at addressing these concerns. In this section we study additional controls related to both episode-specific to country-specific characteristics. The following list describes each control:

- Depth of the recession episode (Δ_{PTy}). Defined as the log change in GDP per capita from output peak to trough. Data source: WEO and WDI.
- Country’s financial development (*fin_development*). Defined as the country’s historical median (1980–2007) of the ratio of bank-provided domestic credit and GDP. Data source: WDI.
- Country size (*small_country*, *medium_country* and *large_country*). Defined as three dummy variables measuring the size of the population of a given country: *small_country* takes the value one when the country’s population is below 20 million and zero otherwise; *medium_country* takes the value one when the country’s population is between 20 and 80 million and zero otherwise; *large_country* takes the value one when the country’s population is above 80 million and zero otherwise. Definition of thresholds and data source: [Uribe and Schmitt-Grohe, 2014](#).
- Country fixed effects. This analysis is only carried out for DMs. For EMs, the use of fixed effects is problematic as the number of countries in the sample is

⁹[Blanchard and Summer \(1986\)](#) depicted the European experience as reflecting hysteresis in unemployment, a situation in which the natural rate of unemployment depends on the actual rate of unemployment. See also [Ball \(2009\)](#).

too large in relation to the overall sample, given by the number of recession episodes.

- Additional labor-market controls. For DMs, we can use an additional set of labor-market controls: those constructed by the OECD, which have been used in the empirical literature as determinants of unemployment rates across countries (see, for example, [Scarpetta, 1996](#)). In particular, we use unemployment benefits (*ub*), the coverage of collective bargaining (*colcov*), and the degree of unionization of the labor force (*union*).

Tables [2.5](#) and [2.6](#) report the estimated coefficient associated with financial crises in Equation [\(2.1\)](#) including these additional controls. The results indicate that there is little change in this association between financial-crisis jobless and wageless recoveries after the inclusion of these variables.

2.4.3 Alternative Measures of Jobless Recoveries

The jobless-recovery measure used in the baseline specification is the change in the unemployment rate from output peak to recovery. This section studies the robustness of the results to two possible concerns related to this measure. A first concern might be that the measure is influenced by a low cyclical rate of unemployment at the output peak. To address this concern, we construct an alternative measure of jobless recovery, defined as the difference between the unemployment rate at the recovery point and the natural rate of unemployment at the output peak ($u_R - \text{natural_}u_P$). A second concern might be that the unemployment rate could also be influenced by changes in the participation rate. To address this concern, we construct an alternative measure of jobless recovery defined as the change in the employment rate between output peak and recovery ($\Delta_{PR}l$).

We estimate Equation [\(2.1\)](#) with these two alternative measures of jobless recov-

TABLE 2.5
FINANCIAL CRISES AND LABOR MARKET RECOVERY—ADDITIONAL CONTROLS—
DEVELOPED MARKET ECONOMIES

Dependent Variable:	$\Delta_{PR}u$				$\Delta_{PR}w$			
Additional Control:	OLS (1)	OLS (2)	IV (3)	IV (4)	OLS (5)	OLS (6)	IV (7)	IV (8)
Panel A: Developed Market Economies								
$\Delta_{PR}y$	0.018*** (0.005)	0.018*** (0.005)	0.025** (0.011)	0.023** (0.011)	-0.043 (0.052)	-0.041 (0.048)	-0.165 (0.158)	-0.168 (0.164)
Observations	45	44	45	44	35	34	35	34
<i>fin_development</i>	0.017*** (0.005)	0.018*** (0.005)	0.026** (0.013)	0.028** (0.014)	-0.041 (0.053)	-0.046 (0.049)	-0.126 (0.159)	-0.156 (0.166)
Observations	45	44	45	44	35	34	35	34
<i>country_size</i>	0.018*** (0.005)	0.019*** (0.005)	0.027** (0.012)	0.027** (0.011)	-0.042 (0.054)	-0.042 (0.050)	-0.138 (0.153)	-0.153 (0.142)
Observations	45	44	45	44	35	34	35	34
Country FE	0.016*** (0.006)	0.015** (0.005)	0.024 (0.016)	0.027* (0.015)	-0.023 (0.053)	-0.019 (0.054)	-0.088 (0.154)	-0.098 (0.134)
Observations	45	44	45	44	35	34	35	34
<i>ub</i>	0.018*** (0.005)	0.019*** (0.005)	0.027** (0.012)	0.027** (0.012)	-0.039 (0.053)	-0.036 (0.048)	-0.118 (0.156)	-0.137 (0.161)
Observations	42	41	42	41	35	34	35	34
<i>colcov</i>	0.019*** (0.005)	0.018*** (0.005)	0.027* (0.013)	0.028** (0.013)	-0.042 (0.053)	-0.028 (0.048)	-0.115 (0.149)	-0.157 (0.155)
Observations	38	37	38	37	35	34	35	34
<i>union</i>	0.014*** (0.005)	0.015*** (0.005)	0.019 (0.012)	0.020 (0.013)	-0.026 (0.053)	-0.039 (0.049)	-0.063 (0.163)	-0.103 (0.165)
Observations	37	36	37	36	34	33	34	33
Other Controls Included								
<i>natural_up</i>	Y	N	Y	N	Y	N	Y	N
<i>lamrig_P</i>	N	Y	N	Y	N	Y	N	Y
<i>gd</i>	Y	Y	Y	Y	Y	Y	Y	Y

Notes: Each coefficient comes from a different regression. Standard errors in parentheses. * indicates significance at 10 percent level; ** at 5 percent level; *** at 1 percent level. Sample and variables definition are detailed in Section 2.2 and 2.4.

TABLE 2.6

FINANCIAL CRISES AND LABOR MARKET RECOVERY—ADDITIONAL CONTROLS
– EMERGING MARKET ECONOMIES

Dependent Variable:	Δ_{PRu}				Δ_{PRw}			
Additional Control:	OLS (1)	OLS (2)	IV (3)	IV (4)	OLS (5)	OLS (6)	IV (7)	IV (8)
Panel B: Emerging Market Economies – Low Inflation								
Δ_{PRy}	0.023** (0.010)	0.020* (0.010)	0.070* (0.036)	0.054* (0.030)	−0.001 (0.103)	−0.037 (0.079)	−0.299 (0.315)	−0.193 (0.217)
Observations	23	23	23	23	21	27	21	27
<i>fin_development</i>	0.027*** (0.009)	0.022** (0.010)	0.061** (0.026)	0.045* (0.022)	0.076 (0.124)	−0.014 (0.090)	−0.433 (0.478)	−0.307 (0.301)
Observations	23	23	23	23	21	27	21	27
<i>country_size</i>	0.021** (0.010)	0.016 (0.011)	0.021* (0.010)	0.018 (0.011)	0.017 (0.100)	−0.007 (0.087)	−0.343 (0.370)	−0.206 (0.365)
Observations	23	23	23	23	21	27	21	27
Panel C: Emerging Market Economies – High Inflation								
Δ_{PRy}	0.011 (0.015)	0.011 (0.015)	0.026 (0.027)	0.026 (0.026)	−0.229* (0.119)	−0.226* (0.130)	−0.380 (0.224)	−0.375 (0.237)
Observations	35	35	35	35	33	33	33	33
<i>fin_development</i>	0.012 (0.014)	0.012 (0.014)	0.021 (0.024)	0.020 (0.022)	−0.251** (0.115)	−0.246** (0.119)	−0.369* (0.194)	−0.327* (0.187)
Observations	35	35	35	35	33	33	33	33
<i>country_size</i>	0.010 (0.015)	0.009 (0.016)	0.025 (0.027)	0.024 (0.026)	−0.288** (0.117)	−0.278** (0.117)	−0.377* (0.200)	−0.371* (0.193)
Observations	35	35	35	35	33	33	33	33
Other Controls Included								
<i>natural_up</i>	Y	N	Y	N	Y	N	Y	N
<i>lamrig_P</i>	N	Y	N	Y	N	Y	N	Y
<i>gd</i>	Y	Y	Y	Y	Y	Y	Y	Y

Notes: Each coefficient comes from a different regression. Standard errors in parentheses. * indicates significance at 10 percent level; ** at 5 percent level; *** at 1 percent level. Sample and variables definition are detailed in Section 2.2 and 2.4.

ery for DMs, in which we have more data coverage on the natural unemployment and employment rates. The results are presented in Table 2.7 and confirm the findings that emerged from the analysis with the change in unemployment rate as a jobless-recovery measure, suggesting that jobless recoveries are not driven by a low unemployment rate at the output peak or by changes in the participation rate.

It is worth noting that the measure of jobless recovery used in this paper differs from that used in other studies that define a jobless recovery as a deviation from the

TABLE 2.7

FINANCIAL CRISES AND ALTERNATIVE MEASURES OF JOBLESS RECOVERY, DEVELOPED MARKET ECONOMIES

Dependent Variable:	OLS (1)	OLS (2)	IV (3)	IV (4)
$\Delta_{PR}u$	0.018*** (0.005)	0.019*** (0.005)	0.028** (0.011)	0.027** (0.011)
Observations	45	44	45	44
$u_R - natural_u_P$	0.021*** (0.005)	0.020*** (0.005)	0.021* (0.011)	0.024* (0.012)
Observations	45	44	45	44
$\Delta_{PR}natural_u_P$	0.012** (0.005)	0.011** (0.005)	0.017 (0.011)	0.020* (0.012)
Observations	45	44	45	44
$u_R - natural_u_R$	0.009** (0.004)	0.009** (0.003)	0.003 (0.008)	0.004 (0.008)
Observations	45	44	45	44
$\Delta_{PR}l$	-0.016*** (0.006)	-0.016*** (0.006)	-0.018 (0.012)	-0.017 (0.012)
Observations	39	38	39	38
Other Controls Included				
$natural_u_P$	Y	N	Y	N
$lamrig_P$	N	Y	N	Y
gd	Y	Y	Y	Y

Notes: Each coefficient comes from a different regression. Standard errors in parentheses. * indicates significance at 10 percent level; ** at 5 percent level; *** at 1 percent level. Sample and variables definition are detailed in Sections 2.2 and 2.4.

Okun’s law, the difference between the unemployment rate and the natural unemployment rate (see, for example, [Ball, Leigh and Loungani, 2013](#)). To compare our findings with these studies, we decompose our jobless-recovery measure into two components: i) the deviation from Okun’s law at the recovery point ($u_R - natural_u_R$), and ii) the change in the natural unemployment rate between output peak and recovery point ($\Delta_{PR}natural_u$). We estimate Equation (2.1) for each of these measures. Results are presented in Table 2.7 and indicate that the major part of the effect found in this paper is driven by changes in the natural rate of unemployment. These changes would not be captured in studies that focus only on deviations from Okun’s law. Nevertheless, our evidence that high-inflation recession episodes do not display jobless recoveries suggests that, more than an increase in the “natural rate,” this

pattern might be better characterized as persistent unemployment in the presence of low inflation and nominal rigidities.

2.5 Financial Crises and Jobless Recoveries: A Simple Analytical Framework

This section develops a simple analytical framework to help explain why financial crises are associated with jobless or wageless recoveries. This framework is based on two observations. The first observation, widely documented in the literature, is that financial crises typically impact collateral values (e.g., a fall in housing prices), tightening credit for firms. For our sample of recession episodes, this is documented in Figure 2.4. During financial-crisis episodes, real house prices contract 8.1 percent from output peak to trough and do not recover once output recovers its trend. In other recession episodes, real house prices only contract 0.8 percent and recover their precrisis level together with output.

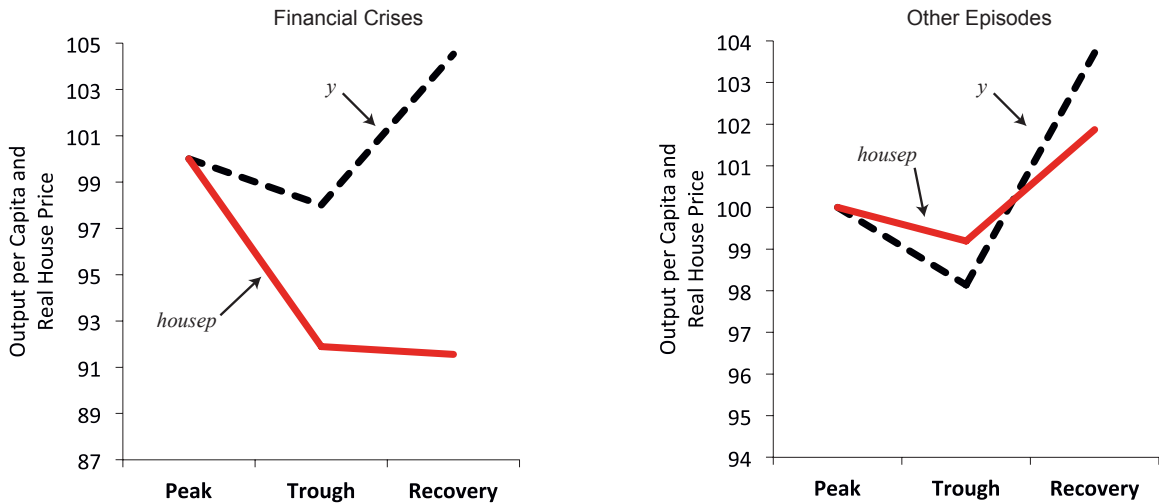


FIGURE 2.4: House Prices and Financial Crises: Developed Market Economies

Notes: See Section 2.2 for a description of the sample and data; y refers to real GDP per capita, $housep$ refers to real house prices, $peak = 100$. Data on real house prices obtained from Cesa-Bianchi, Cespedes and Rebucci (2014).

The second observation that motivates our analytical framework is that not all firms' projects require the same collateral per unit cost. Collateral requirements are lower for projects and firms possessing easily recognizable collateral (e.g., tangible assets) or "intrinsic collateral" (Calvo, 2011). As a large component of such intrinsic collateral is given by physical capital, tighter lending conditions might imply that credit is directed more towards projects that involve physical capital at the expense of projects involving job creation, thus reducing the labor intensity of aggregate output.

In the rest of this section, we begin by formalizing this hypothesis and then provide some empirical evidence on the suggested channel. The model is based on a collateral channel, although it is conceivable that other specifications of the credit market could lead to similar conclusions.

2.5.1 Analytical Framework

Consider a firm that produces homogeneous output by means of capital (K) and labor (L). The production function is denoted by $AF(K, L)$, where A stands for neutral technical progress, and function F displays positive marginal productivities and strictly convex isoquants; F is linear homogenous and twice continuously differentiable. Factors of production have to be hired a period in advance for which credit is required. Therefore, assuming that capital is fully depreciated at the end of the period, and the relevant rate of interest is zero (assumptions that can be relaxed without affecting the central results), profits are given by

$$AF(K, L) - (K + WL), \quad (2.3)$$

where W stands for the wage rate plus search and other costs associated with labor hiring (measured in terms of output).

We assume that firms face a credit constraint,

$$K + WL \leq Z + (1 - \theta)K, \quad (2.4)$$

where $Z > 0$ and $0 \leq \theta < 1$. The left-hand side of Expression (2.4) corresponds to credit needs, while the right-hand side stands for total collateral. Total collateral consists of “extrinsic collateral,” Z , defined as collateral provided by assets other than those involved in the project, and “intrinsic collateral,” $(1 - \theta)K$, defined as the collateral embodied in the project. This helps to capture a situation in which, under credit constraints, capital may be easier to finance than labor. If loans are not repaid, $(1 - \theta)K$ can still be recovered by the creditors. In contrast, funds spent hiring labor cannot be recovered from the workers (unless somebody more skillful than Shylock is involved in the deal!). If K is its own collateral, for example, $\theta = 0$, then this constraint boils down to $wL \leq Z$: labor would be the only input subject to a credit constraint, and capital could be accumulated in the standard manner.

This form of collateral constraint is related to the literature on the *inalienability of human capital* (Hart and Moore, 1994). In this framework, entrepreneurs cannot be costlessly replaced and can repudiate contracts by withdrawing their human capital. It is also related to the literature on *asset tangibility*. For example, Almeida and Campello (2007) show that pledgeable assets support more borrowing because such assets mitigate contractibility problems: Tangibility increases the value that can be captured by creditors in default states. Tangibility as a characteristic of assets used as collateral in debt contracts plays a central role in the corporate finance literature (Tirole, 2005).

The firm’s problem is to choose K and L to maximize (2.3) subject to (2.4). Denoting with λ the Lagrange multiplier associated with the credit constraint (2.4), the optimality conditions are given by (2.4), the first-order conditions,

$$AF_K(K, L) = 1 + \lambda\theta, \quad (2.5)$$

$$AF_L(K, L) = W(1 + \lambda), \quad (2.6)$$

and the complementary slackness conditions,

$$\lambda \geq 0, \quad \lambda(Z - \theta K - WL) = 0. \quad (2.7)$$

Conceivably, Z is determined by the amount of collateral that the firm can credibly post, in addition to capital. A financial crisis can be modeled in this context as a contraction in Z that triggers binding credit constraints. Proposition 2 shows that in this environment, under binding collateral constraints and for a given Z and W , the profit-maximizing technology becomes more capital intensive as A increases.

Proposition 2. *Around a solution of the firms' problem (K^*, L^*) in which credit constraint (2.4) is strictly binding ($\lambda > 0$), $\frac{\partial K^*}{\partial A} > \frac{\partial L^*}{\partial A}$.*

Proof: See Appendix B.2.

This means that output and capital will grow faster than employment. Employment will lag behind output, which is the defining characteristic of a *jobless recovery*.

Figure 2.5 illustrates Proposition 2. As in Proposition 2, we focus on the case in which the credit constraint is strictly binding. The straight line in blue stands for the credit constraint (2.4), whose slope is given by $-\theta/W$. The convex curves are isoprofit lines. Under these conditions, recalling linear homogeneity, one can show that the isoprofit lines in the (L, K) plane are strictly convex, and have the same slope along constant- L/K rays from the origin. Solid and dashed lines correspond to two different families of isoprofit lines. An increase in the neutral technical progress parameter, A , implies that the isoprofit line becomes steeper,¹⁰ and thus an increase in A is equivalent to a shift from the solid to the dashed isoprofit lines. Equilibrium under the solid lines holds at the blue tangent point, while that under the dashed

¹⁰By conditions (2.5) and (2.6), on a given isoprofit line $\frac{\partial L}{\partial K} = -\frac{AF_K(K,L)-1}{AF_L-W} < 0$. This means that $\text{sign } \frac{\partial \frac{\partial L}{\partial K}}{\partial A} = \text{sign} [F_L \frac{\theta}{W} - F_K]$. Combining conditions (2.5) and (2.6), if $\theta < 1$, $F_L \frac{\theta}{W} = \frac{1+\lambda}{\theta+\lambda} F_K < F_K$ and thus $\text{sign } \frac{\partial \frac{\partial L}{\partial K}}{\partial A} < 0$, implying that the isoprofit lines in Figure 2.4 become steeper as A increases.

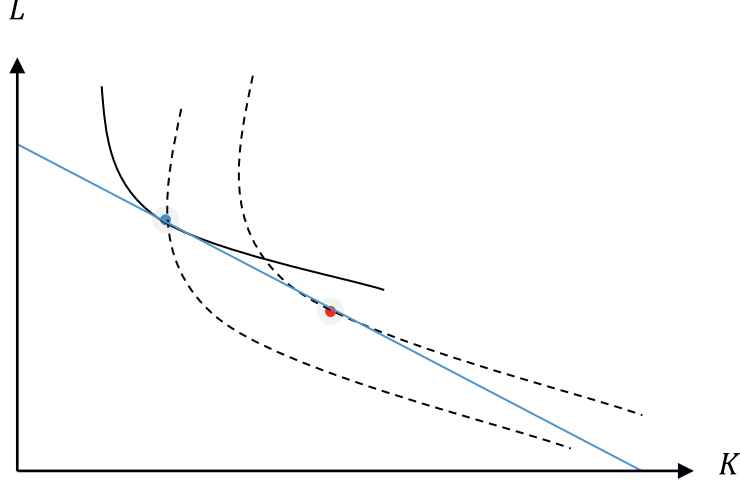


FIGURE 2.5: Optimal Input Vector under Credit Constraint

Notes: Blue line depicts the credit constraint (2.4), black curves are isoprofit lines. Solid and dashed lines correspond to two different families of isoprofit lines; an increase in A is equivalent to a shift from the solid to the dashed isoprofit lines.

lines holds at the red point. Therefore, under binding credit constraints, an increase in A implies an increase in the capital-to-labor ratio.

Although a quantitative study including the mechanism suggested in this section is beyond the scope of this paper, Appendix B.3 presents a numerical experiment using the analytical framework presented in this section and shows that the model can predict a jobless recovery in line with the one observed in the data for the US Great Recession.

2.5.2 Some Empirical Evidence on the Collateral Channel

To further study the transmission mechanism of the analytical framework presented in the previous section, we relate jobless recoveries to the contraction in collateral values, using data on real house prices as proxies for collateral values. These data on real house prices were obtained from Cesa-Bianchi, Céspedes and Rebucci (2014). We estimate an equation similar to (2.1):

$$\Delta_{\text{PR}} z_i = \alpha + \beta \Delta_{\text{PT}} \text{house_} p_i + X_i' \gamma + \epsilon_i. \quad (2.8)$$

where $\Delta_{PT}house_p_i$ denotes the change in (log) real house prices from output peak to trough for recession episode i . Due to data availability, we provide evidence only for the DM sample. Table 2.8 presents results and suggests a negative relationship between the house-price contraction from output peak to trough and jobless recoveries. This result holds for all specifications, using baseline controls and additional controls (Section 2.4), such as country fixed effects.

TABLE 2.8
COLLATERAL VALUES AND JOBLESS RECOVERY DEVELOPED MARKET ECONOMIES

Dependent Variable:		$\Delta_{PR}u$ (Estimation Method: OLS)															
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	
$\Delta_{PR}house_p$	-0.052** (0.021)	-0.054** (0.021)	-0.051** (0.023)	-0.049** (0.023)	-0.049** (0.020)	-0.053** (0.020)	-0.057** (0.021)	-0.063*** (0.021)	-0.049** (0.021)	-0.052** (0.021)	-0.054** (0.022)	-0.052** (0.021)	-0.043** (0.018)	-0.043** (0.019)	-0.052** (0.021)	-0.054*** (0.021)	
$natural_u_P$	0.023 (0.093)		0.023 (0.094)		-0.024 (0.095)		-0.110 (0.124)		0.027 (0.093)		0.031 (0.097)		0.096 (0.083)		0.023 (0.093)		
$lamrig_P$		0.004 (0.003)		0.005 (0.004)		0.003 (0.003)		0.005 (0.003)		0.003 (0.003)		0.006 (0.005)		0.001 (0.003)		0.004 (0.003)	
gd	0.162* (0.084)	0.147* (0.074)	0.165* (0.090)	0.156** (0.076)	0.172** (0.082)	0.132* (0.074)	0.209** (0.089)	0.099 (0.079)	0.142 (0.088)	0.141* (0.076)	0.146 (0.088)	0.122 (0.080)	0.016 (0.079)	0.071 (0.073)	0.162* (0.084)	0.147** (0.074)	
$\Delta_{PR}y$			-0.009 (0.089)	-0.061 (0.095)													
$fin_development$					0.000* (0.000)	0.000 (0.000)											
$small_country$							-0.010 (0.009)	-0.009 (0.008)									
$medium_country$							0.005 (0.007)	0.001 (0.007)									
ub									0.000 (0.000)	0.000 (0.000)							
$colcov$											0.000 (0.000)	0.000 (0.000)					
$union$													0.000 (0.000)	0.000 (0.000)			
Country FE	N	N	N	N	N	N	N	N	N	N	N	N	N	N	Y	Y	
Observations	38	37	38	37	38	37	38	37	38	37	36	35	35	34	38	37	

Notes: Standard errors in parentheses. * indicates significance at 10 percent level; ** at 5 percent level; *** at 1 percent level. Sample and variables definition are detailed in Sections 2.2 and 2.5.

2.6 Conclusions

Financial crises are associated with bad labor-market outcomes. This is a central piece of evidence, which this paper shows for both DMs and EMs. An equally important piece of evidence is that the relationship between financial crises and labor-market outcomes depends on the inflation during the crisis episode. In low-inflation cases (all DMs cases and EMs cases that exhibit inflation below 30 percent annual rate during the recession episode), real wages appear to be downward inflexible, and the brunt of the adjustment comes in the form of high unemployment, measured at the point at which per-capita output recovers its trend. In contrast, under high inflation (EMs cases that exhibit inflation above 30 percent annual rate during the recession episode), unemployment goes back to precrisis levels at the output-recovery point, but real wages are significantly lower.

This suggests that labor-market outcomes during financial crises cannot easily be alleviated by standard expansionary monetary policy. For instance, the evidence suggests that a sharp rise in the price level can help to restore full employment, but at the expense of sharply lower real wages (close to -13 percent according to the average in high-inflation EMs; see Figure 2.2). This indicates that the use of monetary expansion to palliate high unemployment may encounter severe political opposition. Moreover, the EM experience is not helpful to assess the political feasibility in DMs because high inflation was an inevitable consequence of capital flight and resultant maxi-devaluations, not a calculated policy outcome. It is worth noting, incidentally, that there is no evidence in our sample that *persistent inflation* helps to lower unemployment (see Calvo, Coricelli and Ottonello, 2014). In the majority of high-inflation episodes, they occurred mostly within the crisis window and were followed by a return to previous inflation rates. Therefore, the evidence in no way contradicts the vertical Phillips curve conjecture.

Financial-crisis episodes are dramatic events that involve the central nervous sys-

tem of capitalist economies. Hence, there are strong a priori intuitive considerations that make one expect those crises to be deeper and longer than most of the others. It is much less obvious why the labor market should suffer a significantly more powerful blow. To address this issue, the paper presents a simple model in which the financial shock takes the form of a drop in loan collateral values, and firms are assumed to be subject to a binding collateral constraint. This is a standard assumption in the macroeconomic literature (see, for example, [Brunnermeier, Eisenbach and Sannikov, 2012](#)). The relatively new twist in the model is that it assumes that labor costs are harder to collateralize than physical capital because, as a general rule, a share of physical capital can be attached by the creditor in case of default, while hiring costs, for example, are more like “autumn leaves,” hard to grab and harder to price. This slants credit in favor of capital-intensive projects and exacerbates a jobless or wageless recovery. Preliminary tests of this conjecture are encouraging.

The additional evidence about the role of loan collateral further supports the view that standard fiscal and monetary policies may be ineffective in speeding full recovery and suggests that studying policies that address the weaknesses of the credit market should take center stage. Examples include debt restructuring and labor subsidies. Searching for policies of this kind that are both effective and politically viable should be at the top of the policy research agenda to provide guidance to policy intervention during financial crises.

Optimal Exchange-Rate Policy Under Collateral Constraints and Wage Rigidity

Pablo Ottonello

3.1 Introduction

During external crises, exchange-rate policy in emerging market economies (EMs) seem to leave policymakers between a rock and a hard place: Preventing currency depreciation could bring more unemployment, but if liabilities are denominated in foreign currency, currency depreciation could increase debt in terms of domestic income, leading to financial destabilization, and compromising credit access. The potential conflict for exchange-rate policy between these two welfare concerns, credit access and unemployment, is often a central element of the policy debate, as was observed during the East Asian and Latin American crises in the late 1990s ([Fischer, 1998](#); [Calvo, 2001](#); [Stiglitz, 2002](#)) and during the peripheral European crises that started in 2008 (see, for example, [Krugman, 2010](#); [Feldstein, 2011](#)).

This paper conducts a quantitative analysis of the optimal exchange-rate policy when facing this “credit access–unemployment” trade-off. It constructs an environment that provides a theoretical justification for this trade-off, combining two frictions that have been largely studied in the literature: a downward nominal wage rigidity (as in [Schmitt-Grohe and Uribe, 2011](#)), and a financial friction by which external borrowing is denominated in the international unit of account and limited by the

value of collateral in the form of tradable and nontradable income (as in [Mendoza, 2002](#)). In this framework, credit-access and unemployment are two conflicting factors affecting welfare: Devaluations are associated with a welfare gain – by decreasing the real value of wages they reduce involuntary unemployment – but are also associated with a welfare cost – by increasing the value of external debt in terms of domestic income, they tighten the collateral constraint and can trigger an endogenous “sudden stop.”¹

The main finding, in calibrated versions of the model, is that two features characterize the optimal exchange-rate policy during financial crises (defined as episodes of binding credit constraints). First, the optimal allocation generally implies a large real exchange rate depreciation (between a 17 and 40 percent fall on average in the relative price of nontradables), which is achieved by allowing for nominal currency depreciation. The reason is that, the welfare costs related to higher unemployment and lower consumption are typically higher than the welfare costs related to intertemporal misallocation of consumption. Second, optimal currency depreciation is generally lower than that associated with full employment. Thus, the optimal policy is consistent with a managed-floating exchange-rate policy, frequently observed in EMs during financial crises (see, for example, [Calvo and Reinhart, 2002](#)). Moreover, the real exchange-rate depreciation and current account adjustment under the optimal exchange rate policy during episodes of binding collateral constraints is in line with the dynamics observed in the data during sudden stops. The paper shows that the nature of the shocks and the structural characteristics of the economy are key determinants for the optimal degree of “fear of floating” during financial crises: Higher external interest rates, a larger intertemporal or intratemporal elasticity of substitution, or a large mobility of labor across sectors, call for a smaller unemployment; a

¹Calvo (1998) labeled sudden stops episodes of large and abrupt reversals in external credit flows that characterize EMs. For a review of the Fisherian debt-deflation approach to sudden stops, including the form of collateral constraint used in this paper, see [Korinek and Mendoza \(2013\)](#).

more elastic labor supply, or a higher share of income that can be used as collateral, call for more contained currency depreciation.

Welfare under the optimal exchange-rate policy is compared to that under a full-employment and a fixed exchange-rate regimes. The full-employment exchange-rate regime is costly in terms of welfare for making consumption adjust more than what is optimal during periods of binding collateral constraints. The fixed exchange-rate regime is costly for inducing an inefficient adjustment to negative shocks with involuntary unemployment, in periods of both nonbinding and binding collateral constraints. This different nature of welfare costs generally makes the fixed exchange-rate regime more costly, in terms of welfare, than the full-employment exchange-rate regime. The welfare cost of the full-employment and fixed exchange-rate regimes, with respect to the optimal exchange-rate policy, are larger in regions of the state-space where the collateral constraint binds, with an average welfare cost during periods of binding collateral constraints of 0.06 percent and 1.8 percent of consumption per period, respectively.

This is the first paper that conducts a quantitative study of *nominal* exchange-rate policy under a collateral constraint by which debt is limited by the value of collateral in the form of tradable and nontradable income. Introduced in [Mendoza \(2002\)](#), this form of financial friction has been widely used to capture the main stylized facts about sudden stops in EMs.² This form of collateral constraint causes endogenous sudden stops through [Fisher's \(1933\)](#) debt-deflation mechanism: Binding constraints lead to deleveraging, which leads to a fall in the price of nontradables, which further tightens the collateral constraint. Previous studies using this form of financial friction have considered *real* models, in which the policy instrument during periods of binding collateral constraints is a tax (subsidy) on nontradable or tradable

²See, for example, [Mendoza \(2005\)](#), [Durdu, Mendoza, and Terrones \(2009\)](#), [Korinek \(2011\)](#), [Bianchi \(2011\)](#), [Benigno et al. \(2011,2012a,b,c\)](#).

goods (see, for example, [Benigno et al., 2012a](#)). The present paper expands this literature by considering a nominal model and a monetary instrument, which present the policymaker a different trade-off: While subsidizing nontradable goods leads simultaneously to increased employment and higher prices of nontradable goods (relaxing the credit constraint), currency depreciation leads to an increase in employment and a decrease in the price of nontradable goods (tightening the credit constraint).³

The paper is related to the large body of literature that studies nominal exchange-rate policy in small open economies during financial crises. A key difference with respect to this literature is the form of financial friction studied in the present paper, which in turn leads to different policy implications. For instance, in a large subset of this literature, borrowing access is linked to asset prices: [Céspedes, Chang, and Velasco \(2004\)](#), [Devereux, Lane, and Xu \(2006\)](#), [Curdia \(2007\)](#), and [Gertler, Gilchrist, and Natalucci \(2007\)](#) study economies featuring the financial accelerator mechanism (see [Bernanke, Gertler and Gilchrist, 1999](#)). In a recent related paper, [Fornaro \(2013\)](#) study an economy with a collateral constraint that limits external debt to a fraction of the market value of asset holdings (as in [Bianchi and Mendoza, 2011](#)). In these frameworks, currency depreciations have a positive effect on output, that leads to higher asset prices and improved credit access. As a consequence, contrary to the present paper’s result, in these papers flexible exchange rates lead to more financial stability during crises than fixed exchange rates. In the present setup, borrowing access is linked to goods prices: Debt denominated in a foreign currency is limited by the market value of tradable and nontradable income. The combination of a nontradable sector and liability dollarization creates a currency mismatch that makes currency depreciation financially destabilizing (in line with the traditional “original

³In particular, it can be shown that in the model economy presented in Section 3.2, using taxes on nontradable or tradable consumption together with the capital-control tax, a Ramsey planner can achieve an allocation characterized by full employment and nonbinding collateral constraint in all states. See [Benigno et al. \(2012b\)](#) for a similar result in an economy without wage rigidity.

sin” argument; see [Eichengreen, Hausmann, and Panizza, 2005](#); [Calvo, 1999](#)).

The hypothesis of currency depreciations being financially destabilizing has been previously formalized, for instance, in [Aghion, Bacchetta, and Banerjee \(2001\)](#), and in [Braggion, Christiano, and Roldos \(2009\)](#) using credit constraints on firms. In these papers, however, currency depreciations are financially destabilizing because they cause output contraction. In the present paper, currency depreciations are not contractionary (they reduce unemployment), but the associated currency mismatch reduces the *value* of income, leading to a large consumption adjustment under binding credit constraints and entailing a welfare cost. For these reasons, the form of financial friction considered in this paper gives rise to a trade-off between credit access and unemployment that has not been formally studied in the literature of exchange-rate policy in small open economies during financial crises.

The policy choice under the trade-off studied in this paper has engendered a long-standing and still lively policy debate. Keynes, for instance, was first actively opposed to the return of Britain to the gold standard after World War I, arguing that it would be associated with high unemployment ([Keynes, 1925](#)). However, when the Great Depression started, Keynes recommended against devaluation, claiming that now the costs in terms of debt revaluation and financial destabilization would outweigh the benefits ([Irwin, 2011](#)). In the same line, [Diaz-Alejandro \(1965\)](#), analyzing Argentina’s exchange-rate policy in the 1950s, highlighted the possibility that devaluations would lead to negative wealth effects and adjustment in consumption from income distribution and balance-sheet effects. This policy debate was triggered again by the crisis in peripheral Europe that started in 2008, in which there are, simultaneously, high unemployment and high debt levels denominated in euros. Moreover, the empirical literature suggests that both sides of the debate are supported by evidence. Cross-country regressions for EMs tend to show both that fixing the exchange rate during financial crisis episodes is associated with larger output contractions (see, for exam-

ple, [Ortiz et al., 2009](#)), and that currency mismatch plays a key role in determining the access to international credit markets (see, for example, [Calvo, Izquierdo, and Mejia, 2008](#)).

Finally, it is worth noting that while most of the above-mentioned literature on nominal exchange rate policy in small open economies during financial crises compare different (possibly nonoptimal) exchange-rate regimes, the present paper derives the fully optimal exchange-rate policy.⁴ The paper shows that the optimal allocation is a nonmonotonic function of the states; therefore, considering the optimal policy, instead of comparing exchange-rate regimes, is relevant.

The rest of the paper is organized as follows. Section [3.2](#) presents the model economy. Section [3.3](#) defines three possible exchange-rate regimes in this setup (optimal, full-employment, and fixed exchange-rate policies) and provides analytical results describing the exchange-rate policy trade-off that emerges in this economy. Section [3.4](#) presents the quantitative analysis comparing the aggregate dynamics and welfare under the three exchange-rate regimes. Section [3.5](#) examines the sensitivity of results to different calibrations and changes in the baseline model's assumptions. Section [3.6](#) concludes.

3.2 The Model Economy

This section describes the model economy used to conduct exchange-rate policy analysis. It extends the two-sector (tradable and nontradable), dynamic, stochastic, small open economy model with a downward nominal wage rigidity from [Schmitt-Grohe](#)

⁴Optimal monetary policy has been largely studied in open economies with complete asset markets, and in open economies in which the financial friction is that financial markets are incomplete; see, for example, [Clarida, Gali, and Gertler \(2001\)](#), [Schmitt-Grohe and Uribe \(2001\)](#), [Devereux and Engel \(2003\)](#), [Corsetti and Pesenti \(2005\)](#), [Gali and Monacelli \(2005\)](#), [De Paoli \(2009\)](#), [Corsetti, Dedola, and Leduc \(2010\)](#), [Schmitt-Grohe and Uribe \(2011\)](#). The present paper constitutes a contribution in this direction for a small open economy in which financial frictions include an imperfect access to credit markets, with the presence of occasionally binding collateral constraints.

and Uribe (2011), to include a collateral constraint in the form of tradable and non-tradable income. The economy only has access to a one-period, non-state-contingent debt instrument, denominated in units of tradable goods, capturing liability dollarization. The model then features a nominal rigidity and two financial frictions that will interact to determine the exchange-rate policy trade-off.

Tradables are endowed to the economy, and their price is determined by the law of one price. Nontradables are produced by the economy, and their price is determined by domestic demand and supply. Fluctuations in the small open economy are driven by exogenous shocks to the value of the tradable endowment (which can be interpreted as shocks to terms of trade or to productivity in the tradable sector) and to the interest rate on external debt, two sources of business-cycle fluctuations that have been widely studied in EMs (Mendoza, 1995; Neumeyer and Perri, 2005; Uribe and Yue, 2006).

3.2.1 Households

Households' preferences over consumption are described by the expected utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \quad (3.1)$$

where c_t denotes consumption in period t ; the function $U(\cdot)$ is assumed to be continuous, twice differentiable, strictly increasing, and concave; the subjective discount factor $\beta \in (0, 1)$, and \mathbb{E}_t denotes expectation conditional on the information set available at time t .

The consumption good is assumed to be a composite of tradable and nontradable goods, with a CES aggregation technology:

$$c_t = A(c_t^T, c_t^N) = \left[a(c_t^T)^{1-\frac{1}{\xi}} + (1-a)(c_t^N)^{1-\frac{1}{\xi}} \right]^{\frac{\xi}{\xi-1}}, \quad (3.2)$$

where c_t^T denotes tradable consumption and c_t^N denotes nontradable consumption.

Each period, households receive a stochastic endowment (y_t^T) and profits from the ownership of firms producing nontradable goods (Π_t). They inelastically supply \bar{h} hours of work to the labor market. (Section 3.5 relaxes these assumptions studying production in the tradable sector and an elastic labor supply.) Due to the presence of the wage rigidity (discussed in detail in the next sections), households will only be able to sell $h_t \leq \bar{h}$ hours in the labor market. The level of actual hours worked (h_t) is determined by firms and is taken as given by the households.

Households have access to a one-period, non-state-contingent bond denominated in units of tradable goods that can be traded internationally paying an exogenous and stochastic gross interest rate R_t . The model therefore assumes full liability dollarization. It is assumed that the vector of exogenous states, $s_t^X \equiv [y_t^T, R_t]$, follows a first-order Markov process. Debt acquired in period t is taxed at rate τ_t^d . Households' sequential budget constraint is therefore given by

$$\frac{d_{t+1}}{R_t} (1 - \tau_t^d) = d_t + c_t^T + p_t c_t^N - (y_t^T + w_t h_t + \Pi_t) - T_t, \quad (3.3)$$

where d_{t+1} denotes the level of debt assumed in period t and due in period $t + 1$, $p_t \equiv \frac{P_t^N}{P_t^T}$ denotes the relative price of nontradables in terms of tradables, w_t denotes the wage rate in terms of tradable goods, and T_t denotes a lump sum transfer in period t .

It is assumed that households face a collateral constraint by which external debt cannot exceed a fraction κ of income:

$$d_{t+1} \leq \kappa (y_t^T + w_t h_t + \Pi_t), \quad (3.4)$$

where $\kappa > 0$. This form of collateral constraint, introduced in [Mendoza \(2002\)](#), has been used extensively in the literature on small open economies to capture the effect of currency mismatch on external credit-market access: While collateral includes income from both tradable and nontradable sectors, external debt is fully denominated in

units of tradables. The credit-market frictions from which this constraint arises are not modeled here explicitly, but this form of collateral constraint can be seen as describing an environment in which lenders manage default risk by imposing a debt limit linked to households' current income, as is typically the case of lending criteria in mortgage or consumer credit markets.⁵ Empirical evidence suggests that current income is a significant determinant of credit market access ([Jappelli, 1990](#)).

In addition, households are assumed to face a no-Ponzi game constraint of the form

$$d_{t+1} \leq d^N, \quad (3.5)$$

where d^N denotes the *natural debt limit*. As in [Aiyagari \(1994\)](#), this is defined as the maximum value of external debt that the household can repay almost surely starting from that period, assuming that its tradable consumption is zero forever. Formally, denoting \underline{y}^T as the minimum possible level of tradable endowment and \bar{R} as the maximum possible level of external interest rate, the natural debt limit is defined as $d^N \equiv \frac{\bar{R}}{\bar{R}-1} \underline{y}^T$. Since the collateral value in the credit limit (3.4) depends on relative prices which can be affected by policy variables, constraint (3.5) is imposed in addition to (3.4) in order to prevent Ponzi schemes induced by the policymaker ([Mendoza, 2005](#); [Benigno et al., 2012b](#)).

The household problem is to choose state-contingent plans for c_t , c_t^T , c_t^N , and d_{t+1} that maximize the expected utility (3.1) subject to the consumption aggregation technology (3.2), the sequential budget constraint (3.3), the collateral constraint (3.4), and the no-Ponzi game constraint (3.5), for a given initial debt level, d_0 ; for the given sequence of prices, w_t and p_t ; for the given sequence of hours worked, h_t , profits, Π_t , stochastic tradable endowment, y_t^T , and interest rate, R_t ; and for the given sequence

⁵[Korinek \(2011\)](#) shows that this form of the collateral constraint can be rationalized as a renegotiation-proof form of debt contract in an imperfect credit market in which households can renegotiate external debt and lenders can extract at most a fraction of borrowers' current income if debt is renegotiated.

of policies, τ_t^d and T_t .

Denoting by λ_t the Lagrange multiplier associated with the budget constraint (3.3) and by μ_t the Lagrange multiplier associated with the collateral constraint (3.4), the optimality conditions (provided $d_{t+1} < d^N$) are (3.2), (3.3), and (3.4), with the first-order conditions

$$\lambda_t R_t^{-1} (1 - \tau_t^d) = \beta \mathbb{E}_t \lambda_{t+1} + \mu_t, \quad (3.6)$$

$$U_c A_T (c_t^T, c_t^N) = \lambda_t, \quad (3.7)$$

$$\left(\frac{1-a}{a} \right) \left(\frac{c_t^T}{c_t^N} \right)^{\frac{1}{\xi}} \equiv \mathcal{P} (c_t^T, c_t^N) = p_t, \quad (3.8)$$

and the complementary slackness conditions

$$\mu_t \geq 0, \mu_t (\kappa (y_t^T + w_t h_t + \Pi_t) - d_{t+1}) = 0. \quad (3.9)$$

3.2.2 Firms

Each period, operating in competitive labor and product markets, firms hire labor to produce the nontradable good, y_t^N . Profits each period are given by

$$\Pi_t = p_t F(h_t) - w_t h_t,$$

where the production function, $F(\cdot)$, is assumed to be increasing and concave.

The firms' problem is to choose h_t to maximize profits given prices p_t and w_t . The first-order condition of this problem is

$$p_t F'(h_t) = w_t. \quad (3.10)$$

This condition implicitly defines the firms' demand for labor.

3.2.3 The Labor Market

Nominal wages (W_t) are assumed to be downwardly rigid as in [Schmitt-Grohe and Uribe \(2011\)](#).⁶

$$W_t \geq \gamma W_{t-1},$$

for $\gamma > 0$.

It is assumed that the *law of one price* holds for tradable goods, implying that $P_t^T = E_t P_t^{T*}$, where E_t is the nominal exchange rate and P_t^{T*} is the foreign currency price of tradable goods. Assuming that P_t^{T*} is constant and normalized to one, wages in terms of tradable goods (w_t) can be expressed as

$$w_t = \frac{W_t}{E_t}.$$

From this, the wage rigidity can be expressed as

$$w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t}, \tag{3.11}$$

where ϵ_t is the gross depreciation rate of the nominal exchange rate: $\epsilon_t \equiv \frac{E_t}{E_{t-1}}$.

However, actual hours worked cannot exceed the inelastically supplied level of hours:

$$h_t \leq \bar{h}. \tag{3.12}$$

When the nominal wage rigidity binds, the labor market can exhibit involuntary unemployment, given by $\bar{h} - h_t$. This implies a slackness condition must hold at all dates and states:

$$\left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) (\bar{h} - h_t) = 0. \tag{3.13}$$

⁶The assumption of an asymmetric nominal wage rigidity is consistent with empirical evidence using microeconomic data (e.g., [Gottschalk, 2005](#); [Barattieri, Basu and Gottschalk, 2010](#); [Daly, Hobijn, and Lucking, 2012](#)).

This condition means that when the nominal wage rigidity is not binding, the labor market must exhibit full employment, and if it exhibits unemployment, it must be the case that the nominal wage rigidity is binding.

3.2.4 The Government

The government determines the exchange-rate depreciation, ϵ_t , and imposes a proportional tax (subsidy) on debt τ_t^d , which is rebated lump sum to households (T_t), to balance its budget each period:

$$\frac{d_{t+1}}{R_t} \tau_t^d = T_t. \quad (3.14)$$

Section 3.3 defines different exchange-rate regimes and how the capital-control tax is determined.

3.2.5 General Equilibrium Dynamics

The market for nontradable goods clears at all times:

$$c_t^N = F(h_t). \quad (3.15)$$

Combining the equilibrium price equation, (3.8), with condition (3.15), the firms' optimality condition, (3.10), can be expressed as

$$w_t = \left(\frac{1-a}{a} \right) (c_t^T)^{\frac{1}{\xi}} F(h_t)^{-\frac{1}{\xi}} F'(h_t). \quad (3.16)$$

Combining condition (3.15) with households' budget constraint, (3.3), the definition of firms' profits, and the government's budget constraint, (3.14), the resource constraint of the economy becomes

$$\frac{d_{t+1}}{R_t} = d_t + c_t^T - y_t^T. \quad (3.17)$$

Using the definition of firms' profits, the equilibrium price equation, (3.8), and the market clearing condition for nontradables, (3.15), the collateral constraint, (3.4),

can be reexpressed as

$$d_{t+1} \leq \kappa \left(y_t^T + \left(\frac{1-a}{a} \right) (c_t^T)^{\frac{1}{\xi}} F(h_t)^{\frac{\xi-1}{\xi}} \right) \equiv \bar{d}(h_t, c_t^T, y_t^T). \quad (3.18)$$

The general equilibrium dynamics are then given by stochastic processes $\{c_t^N, c_t^T, h_t, p_t, w_t, d_{t+1}, \lambda_t, \mu_t, T_t\}_{t=0}^{\infty}$ satisfying the set of equations (GE):

$$(3.5): d_{t+1} \leq d^N,$$

$$(3.6): \lambda_t R_t^{-1} (1 - \tau_t^d) = \beta \mathbb{E}_t \lambda_{t+1} + \mu_t,$$

$$(3.7): U_c(c_t^T, c_t^N) A_T(c_t^T, c_t^N) = \lambda_t,$$

$$(3.8): p_t = \mathcal{P}(c_t^T, c_t^N),$$

$$(3.9): \mu_t \geq 0, \mu_t (\kappa (y_t^T + p_t F(h_t)) - d_{t+1}) = 0,$$

$$(3.11): w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t},$$

$$(3.12): h_t \leq \bar{h},$$

$$(3.13): \left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) (\bar{h} - h_t) = 0,$$

$$(3.14): T_t = \tau_t^d d_{t+1} R_t^{-1},$$

$$(3.15): c_t^N = F(h_t),$$

$$(3.16): w_t = \left(\frac{1-a}{a} \right) (c_t^T)^{\frac{1}{\xi}} F(h_t)^{-\frac{1}{\xi}} F'(h_t),$$

$$(3.17): d_{t+1} R_t^{-1} = d_t + c_t^T - y_t^T,$$

$$(3.18): d_{t+1} \leq \kappa \left(y_t^T + \left(\frac{1-a}{a} \right) (c_t^T)^{\frac{1}{\xi}} F(h_t)^{\frac{\xi-1}{\xi}} \right),$$

given an exchange-rate policy $\{\epsilon_t\}_{t=0}^{\infty}$, a capital-control tax policy $\{\tau_t^d\}_{t=0}^{\infty}$, initial conditions w_{-1} and d_0 , and exogenous stochastic processes $\{y_t^T, R_t\}_{t=0}^{\infty}$.

3.3 Exchange-Rate Regimes: Definitions and Analytical Results

This section formally defines the optimal exchange-rate policy, and discusses the trade-off between credit access and unemployment that exchange-rate policy can face in the model economy presented in the previous section. Analytical results relating

credit access and unemployment are established, providing a framework for understanding the quantitative characterization of the optimal exchange-rate policy to be presented in the next section. Two additional exchange-rate regimes are also defined in this section – full-employment and fixed exchange-rate policy – to provide standard benchmarks for the study of the optimal exchange-rate policy.

3.3.1 Definition of Exchange-Rate Regimes

This section defines three possible exchange-rate regimes: optimal, full-employment, and fixed exchange-rate policy. Exchange-rate regimes are defined conditional on an optimal capital-control tax policy (τ_t^d). The reason for using this capital-control tax is twofold. First, previous literature has shown that both the credit constraint and the downward wage rigidity considered in this paper embody a pecuniary externality that may induce inefficient external borrowing (Bianchi, 2011; Beningo et al., 2012a; and Schmitt-Grohe and Uribe, 2013).⁷ The optimal capital-control tax policy eliminates any borrowing inefficiency, and allows for a comparison across exchange-rate regimes isolating the effect of exchange-rate policy from this distortion.

Second, without the optimal capital-control tax, the set of restrictions for the optimal policy includes a forward-looking constraint (namely, the household’s intertemporal borrowing decision (3.6)). As shown in Kydland and Prescott (1977), Bellman’s (1957) principle of optimality fails in this context, and standard dynamic programming techniques cannot be applied. Using an optimal capital-control tax

⁷Inefficient borrowing arises when the social costs of borrowing differ from the private costs of borrowing. Bianchi (2011) shows that in an endowment economy, the collateral constraint in the form of tradable and nontradable income induces overborrowing, in the sense that the social costs of borrowing exceed the private costs of borrowing; in this setup, the constrained social planner borrows less than the competitive equilibrium. Beningo et al. (2011, 2012a) define overborrowing (underborrowing) as a situation in which a constrained social planner would take on less (more) debt than decentralized agents; in this sense, the authors find that whether an economy with this form of collateral constraint features overborrowing or underborrowing depends on the structure of the economy (e.g., endowment or production), and on the calibration. Schmitt-Grohe and Uribe (2013) show that the downward wage rigidity, combined with a fixed exchange-rate policy, induces overborrowing.

technically simplifies the problem, allowing for the use of standard dynamic programming techniques. Nevertheless, Section 3.5 studies the sensitivity of the optimal exchange-rate policy to the assumption of optimal capital-control taxes by restricting the Ramsey planner's set of available instruments to the nominal exchange rate. In this context, time-invariant optimal policies under commitment are obtained using the recursive saddle-point method developed in [Marcet and Marimon \(2011\)](#).

3.3.1.1 Optimal Exchange-Rate Policy

Definition 4. *The optimal exchange-rate policy with optimal capital-control taxes is the set of processes $\{\epsilon_t, \tau_t^d\}$ that maximize households' expected lifetime utility (3.1) subject to the set of equations describing the general equilibrium dynamics (GE).*

To characterize the allocation under the optimal exchange-rate policy with optimal capital-control taxes, I set up the Ramsey problem dropping constraints (3.6)–(3.9), (3.11), and (3.13)–(3.16). Appendix C.1 shows that any $\{d_{t+1}, c_t^T, h_t\}$ that satisfy (3.5), (3.12), (3.17), and (3.18) also satisfy (GE). The Ramsey problem is then to maximize (3.1) with respect to $\{d_{t+1}, c_t^T, h_t\}$, subject to (3.5), (3.12), (3.17), and (3.18). The dynamics under the optimal exchange-rate policy with optimal capital-control taxes can be thus expressed with the Bellman equation,

$$\begin{aligned}
V^{OP}(s^X, d) &= \max_{d', c^T, h} [U(A(c^T, F(h))) + \beta \mathbb{E}_{s^{X'}} V^{OP}(s^{X'}, d')] \quad (3.19) \\
s.t. \quad \frac{d'}{R} &= d + c^T - y^T, \\
d' &\leq \kappa \left(y^T + \left(\frac{1-a}{a} \right) (c^T)^{\frac{1}{\xi}} F(h)^{\frac{\xi-1}{\xi}} \right), \\
d' &\leq d^N, \\
h &\leq \bar{h},
\end{aligned}$$

where time subscripts for variables dated in period t have been dropped, and a prime is used to indicate variables dated in period $t+1$; $V^{OP}(s^X, d)$ denotes the value

function for households under optimal exchange-rate and capital-control tax policies. This formulation will be used in the quantitative analysis.

3.3.1.2 Full-Employment Exchange-Rate Policy

For this regime, consider an exchange-rate policy aimed at maintaining full employment at all states and dates: Under the full-employment policy,

$$h_t = \bar{h}, \quad \forall t. \quad (3.20)$$

Definition 5. *The full-employment exchange-rate policy with optimal capital-control taxes is the set of processes $\{\epsilon_t, \tau_t^d\}$ that maximize households' expected lifetime utility (3.1) subject to the set of equations describing the general equilibrium dynamics (GE), and the full-employment constraint (3.20).*

To characterize the optimal allocation under the full-employment policy, I follow the same strategy as for the optimal exchange-rate policy and drop constraints (3.6)–(3.9) and (3.11)–(3.16). Appendix C.1 shows that any $\{d_{t+1}, c_t^T, h_t\}$ that satisfy (3.5), (3.17), (3.18), and (3.20), also satisfy (GE) and (3.20). Therefore, the dynamics under the full-employment exchange-rate policy with optimal capital-control taxes can be expressed with the Bellman equation,

$$\begin{aligned} V^{FE}(s^X, d) &= \max_{d', c^T} [U(A(c^T, F(\bar{h}))) + \beta \mathbb{E}_{s^X} V^{FE}(s^{X'}, d')] \quad (3.21) \\ \text{s.t. } \frac{d'}{R} &= d + c^T - y^T, \\ d' &\leq \kappa \left(y^T + \left(\frac{1-a}{a} \right) (c^T)^{\frac{1}{\xi}} F(\bar{h})^{\frac{\xi-1}{\xi}} \right), \\ d' &\leq d^N, \end{aligned}$$

where $V^{FE}(s^X, d)$ denotes the value function for households under the full-employment exchange-rate policy with optimal capital-control taxes.

3.3.1.3 Fixed Exchange-Rate Policy

Finally, consider a policy aimed at keeping the exchange rate fixed at all states and dates: Under the fixed exchange-rate policy or currency peg,

$$\epsilon_t = 1, \forall t. \quad (3.22)$$

Definition 6. *The fixed exchange-rate policy with optimal capital-control taxes is the set of processes $\{\epsilon_t, \tau_t^d\}$ that maximize households' expected lifetime utility (3.1) subject to the set of equations describing the general equilibrium dynamics (GE), and currency peg constraint (3.22).*

To characterize the allocation under the currency peg with optimal capital-control taxes, I follow a similar strategy to that of the optimal exchange-rate policy and drop constraints (3.6)–(3.9) and (3.14)–(3.15). Appendix C.1 shows that any $\{d_{t+1}, c_t^T, h_t, w_t, \epsilon_t\}$ that satisfy (3.5), (3.11)–(3.13), (3.16)–(3.18), and (3.22), also satisfy (GE) and (3.22). Thus, the dynamics under the currency peg with optimal capital-control tax policy can be expressed with the Bellman equation,

$$V^{PEG}(s^X, d, w_{-1}) = \max_{d', c^T, h, w} [U(A(c^T, F(h))) + \beta \mathbb{E}_{s^X} V^{PEG}(s^{X'}, d', w)] \quad (3.23)$$

$$\begin{aligned} s.t. \quad & \frac{d'}{R} = d + c^T - y^T, \\ & d' \leq \kappa \left(y^T + \left(\frac{1-a}{a} \right) (c^T)^{\frac{1}{\xi}} F(h)^{\frac{\xi-1}{\xi}} \right), \\ & d' \leq d^N, \\ & w \geq \gamma w_{-1}, \\ & h \leq \bar{h}, \\ & (w - \gamma w_{-1})(\bar{h} - h) = 0, \\ & w = \left(\frac{1-a}{a} \right) (c^T)^{\frac{1}{\xi}} F(h)^{-\frac{1}{\xi}} F'(h), \end{aligned}$$

where $V^{PEG}(s^X, d, w_{-1})$ denotes the value function for households under the currency peg and optimal capital-control taxes and the subscript -1 is used to indicate variables dated in period $t - 1$.

3.3.2 Optimal Exchange-Rate Policy, Unemployment and Credit Limit: Analytical Results

This section studies the relationship between unemployment and the credit limit under the optimal exchange-rate policy. Although, given the complexity of the model, a numerical solution is required for a full characterization, some analytical results can be obtained to show the trade-off involved in exchange-rate policy. These results will be relevant to understanding the next section's numerical solution for the dynamics of the economy under the optimal exchange-rate policy. Proposition 1 characterizes the allocation under the optimal exchange-rate policy defined in the previous section.

Proposition 3. *Under the optimal exchange-rate policy with optimal capital-control taxes (Definition 1) the following conditions hold at all dates and states:*

- If $\xi < 1$, $(\bar{h} - h_t)(\bar{d}(h_t, c_t^T, y_t^T) - d_{t+1}) = 0$.
- If $\xi \geq 1$, $h_t = \bar{h}$.

Proof. See Appendix C.2. ■

Two conclusions follow from this result. First, the allocation under the optimal exchange-rate policy and capital-control taxes features no unemployment under no binding collateral constraints. Given that the capital-control tax eliminates any inefficient borrowing, eliminating unemployment when the credit constraint does not bind leads to a welfare gain (a higher consumption of nontradables), without any welfare cost.⁸

⁸Note that while the presence of incomplete financial markets leads to inefficient consumption fluctuations relative to an economy with complete asset markets, eliminating unemployment does

Second, if the intratemporal elasticity of substitution is less than one, a slackness condition is established between unemployment and the collateral constraint under the optimal exchange-rate policy: If the collateral constraint is not binding, the labor market must exhibit full employment, and if there is unemployment, the collateral constraint must be binding. As discussed at the end of this section, empirical evidence from EMs provides wide support for the intratemporal elasticity of substitution being less than one. To understand the role of the intratemporal elasticity of substitution and the interaction between unemployment and the collateral constraint under the optimal exchange-rate policy, a discussion is in order regarding the trade-off facing exchange-rate policy in this economy.

3.3.2.1 The Credit-Access–Unemployment Trade-off

Parallel to the traditional *inflation–unemployment* trade-off in the New Keynesian literature, the exchange-rate policy in this economy may face a “credit-access–unemployment” trade-off. Under binding nominal downward wage rigidity, a depreciation of the nominal exchange rate decreases real wages and, thus, helps reduce unemployment. But it is also associated with a real exchange-rate depreciation, which decreases the value of nontradable output in tradable units. Recall that the collateral in this economy is given by the value, in tradable units, of tradable and nontradable output. Accordingly, if the price effect (real exchange-rate depreciation) dominates the quantity effect (employment increase), an exchange-rate depreciation can decrease the collateral value and tighten the credit limit. The price effect dominates the quantity effect if the intratemporal elasticity of substitution between tradables and nontradables is less than one ($\xi < 1$). As discussed in the next section, this assumption is widely supported by empirical evidence from EMs. Under this assumption, the following proposition can be established:

not lead to *more* inefficient consumption fluctuations.

Proposition 4. *If $\xi < 1$, given an initial state (s_t^X, d_t) , for any debt level d_{t+1}^* with associated tradable consumption $c_t^{T*} = (d_{t+1}^* R_t^{-1} - d_t + y_t^T) > 0$ for which $d_{t+1}^* > \bar{d}(\bar{h}, c_t^{T*}, y_t^T)$, there exists a level of employment $h_t^* \in (0, \bar{h})$ for which $d_{t+1}^* = \bar{d}(h_t^*, c_t^{T*}, y_t^T)$.*

Proof. See Appendix C.2. ■

This result shows that for any debt level that does not satisfy the credit limit under full employment, there exists a level of employment below full employment for which the real exchange rate is sufficiently appreciated to ensure the credit limit is satisfied for that debt level. This result stems from the fact that if the intratemporal elasticity of substitution is less than one ($\xi < 1$), the collateral constraint is decreasing in the level of employment.⁹ This provides a theoretical justification for the existence of the exchange-rate policy debate, typically observed during financial crises in EMs, that weighs the two policy objectives. The optimal choice under this trade-off can be characterized using the first-order conditions of the optimal policy problem (3.19):

Remark 1 If $\xi < 1$, in an allocation under the optimal exchange-rate policy with optimal capital-control taxes (Definition 1) in which, at time t , $h_t < \bar{h}$, the following conditions hold:

$$U_c A_N(c_t^T, F(h_t)) = \phi_t^\mu \left(\frac{1 - \xi}{\xi} \right) \kappa \mathcal{P}(c_t^T, F(h_t)), \quad (3.24)$$

$$\phi_t^\mu = \left(\frac{\phi_t^F}{R_t} - \beta \mathbb{E}_t \phi_{t+1}^F \right), \quad (3.25)$$

where ϕ_t^μ , and ϕ_t^F denote the nonnegative multipliers associated with the collateral constraint (3.18), and the resource constraint (3.17), respectively, in the Ramsey problem of optimal exchange-rate policy with optimal capital-control taxes.

⁹If the intratemporal elasticity of substitution is greater than or equal to one ($\xi \geq 1$), the credit access–unemployment trade-off vanishes, as implied by Proposition 3. In particular, if the intratemporal elasticity of substitution is *equal* to one ($\xi = 1$), employment does not influence the collateral constraint. If the intratemporal elasticity of substitution is *greater* than one ($\xi > 1$), the credit-access–unemployment trade-off overturns, and a decrease in unemployment also helps relax the collateral constraint.

Proof. See Appendix C.2. ■

Equation (3.24) shows that in any optimal allocation in which there is unemployment, the Ramsey planner equates the marginal benefit of increasing employment, given by the marginal utility of nontradable consumption, to its marginal cost in terms of tightening the collateral constraint. Equation (3.25) shows that the shadow price of relaxing the credit constraint for the Ramsey planner, ϕ_t^μ , is the wedge between the current shadow value of wealth for the Ramsey planner and the expected value of reallocating wealth to the next period. This shows a relevant aspect of the trade-off involved in exchange-rate policy: While the costs of exchange-rate depreciations are associated with intertemporal misallocation of consumption, their benefits are related to a higher level of consumption.

3.3.2.2 Empirical Evidence on the Intratemporal Elasticity of Substitution

As shown in Propositions 3 and 4, a tension exists between credit access and unemployment only if the elasticity of substitution between tradable and nontradable goods is less than one ($\xi < 1$). If this is the case, tradable and nontradable goods are gross complements, and the price effect (real exchange-rate depreciation) associated with increasing employment dominates the quantity effect (employment increase). As a result, exchange-rate depreciation can decrease the collateral value and make the credit limit tighter.

There is wide support from the empirical literature for the intratemporal elasticity of substitution being less than one. In a sample of developed and emerging market economies, Stockman and Tesar (1995) estimate a value of the elasticity of substitution of 0.44. Separating the samples of developed and emerging economies, Mendoza (1995) finds values of the elasticity of 0.74 and 0.43, respectively. In studies for EMs, Gonzalez-Rozada et al. (2004) found estimates in the range between 0.4

and 0.48 for Argentina and [Lorenzo, Aboal, and Osimani \(2005\)](#), found estimates in a range between 0.46 and 0.75 for Uruguay.¹⁰

Moreover, following this empirical literature, the studies referenced in the present paper that calibrate a two-sector, small open economy model generally use a parameter value of the elasticity of substitution in the range between 0.44 and 0.83.

3.4 Quantitative Analysis

The objective of this section is to quantitatively characterize the aggregate dynamics of the model economy under the optimal exchange-rate policy and to compare its performance, in terms of welfare, to that under the full-employment and fixed exchange-rate policies, both during periods of financial crises and under regular business-cycle fluctuations.

3.4.1 Calibration and Computation

To characterize the aggregate dynamics under the different exchange-rate regimes, calibrated versions of the functional equations [\(3.19\)](#), [\(3.21\)](#) and [\(3.23\)](#) are solved numerically. Due to the presence of occasionally binding constraints, I resort to the method of value-function iteration over a discretized state-space to compute the numerical solutions.

As mentioned in [Section 3.2](#), the consumption aggregator is assumed to be a CES aggregator. I also assume a CRRA period utility function and an isoelastic form for

¹⁰[Ostry and Reinhart \(1992\)](#) found evidence inconclusive in this respect with estimates between 0.66 and 1.44, depending on the EM region and the instrumental variable considered. For a survey on the methodologies used to estimate the elasticity of substitution between tradable and nontradable goods, see [Akinci \(2011\)](#).

the production function:

$$\begin{aligned} U(c) &= \frac{c^{1-\sigma} - 1}{1 - \sigma}, \\ F(h) &= h^{\alpha^N}. \end{aligned} \tag{3.26}$$

The model is calibrated at the annual frequency, to match Argentinean data. Argentina is used as a benchmark to conduct this exercise as an EM country whose exchange-rate regimes and financial crises have been widely studied, particularly in the two branches of the literature this paper combines.

All parameter values used in the baseline calibration are shown in Table 3.1. The inverse of the intertemporal elasticity of substitution is set to $\sigma = 2$, a standard value in the business-cycle literature for small open economies (see, for example, [Mendoza 1991](#)). The intratemporal elasticity of substitution is set to $\xi = 0.44$, using the estimates of [Gonzalez-Rozada et al. \(2004\)](#) for Argentina (see Section 3.3.2 for a review of the literature on this parameter). I set $\alpha^N = 0.75$, following the evidence in [Uribe \(1997\)](#) on the labor share in the nontradable sector in Argentina, and $\gamma = 0.96$, following the evidence in [Schmitt-Grohe and Uribe \(2011\)](#) on downward nominal wage rigidity. The mean level of tradable output and the labor endowment (\bar{h}) are normalized to one.

The parameters $\{\beta, a, \kappa\}$ are used to match three key moments in the ergodic distributions of the model under the optimal exchange-rate policy to the ones observed in historical Argentinean data (for the period 1975–2011). The three data moments considered are typically targeted in the related literature (following [Bianchi, 2011](#)): an average level of external debt-to-GDP ratio of 21 percent, a share of tradable output in GDP of 32.9 percent, and a frequency of sudden stops of 5.5 percent. A sudden stop in the model is defined as a period in which the economy exhibits a change in the current account larger than one standard deviation, following [Eichengreen, Gupta and Mody \(2006\)](#), from which the frequency of sudden stops is obtained for a sample

TABLE 3.1
BASELINE PARAMETER VALUES

Parameter	Value	Description
σ	2	Inverse of intertemporal elasticity of substitution
ξ	0.44	Intratemporal elasticity of substitution
a	0.295	Share of tradables
β	0.8	Annual subjective discount factor
κ	0.263	Share of income used as collateral
α^N	0.75	Labor share in nontradable sector
γ	0.96	Degree of downward nominal rigidity

of EMs.¹¹ (See the Data appendix for further details on data sources, and on the construction of the series). The parameter values obtained from this calibration are $\beta = 0.8$, $a = 0.295$ and $\kappa = 0.263$. Section 3.5 studies the sensitivity of the optimal policy to this calibration.

It is assumed that the two exogenous driving forces, the tradable endowment and the interest rate, follow a first-order VAR of the form

$$\begin{bmatrix} \ln(y_t^T) \\ \ln\left(\frac{R_t}{R}\right) \end{bmatrix} = \Phi \begin{bmatrix} \ln(y_{t-1}^T) \\ \ln\left(\frac{R_{t-1}}{R}\right) \end{bmatrix} + \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^R \end{bmatrix}, \quad (3.27)$$

where $\begin{bmatrix} \varepsilon_t^y & \varepsilon_t^R \end{bmatrix}' \sim \text{i.i.d. } N(\emptyset, \Omega)$ and R denotes the mean interest rate level.

The parameters of this stochastic process are estimated using Argentinean data since 1983. Tradable endowment is measured with the cyclical component of value added in agriculture and manufacturing. Interest rates on external debt are measured as the sum of EMBI spreads and the Treasury-bill rate. (Section 3.5 studies an economy with interest rate shocks calibrated to those of the risk-free rate). Since the data on EMBI spreads for Argentina is available since 1994, the series were extended back to 1983, using the Neumeyer and Perri (2005) dataset, which uses a measure similar to the one considered here. The interest rate series is then deflated with a measure of expected dollar inflation. (See the Data appendix for further

¹¹The frequency for EMs is similar to the frequency in Argentina during this period, and to other empirical estimates, such as Calvo et al. (2008).

details on data sources, and on the construction of tradable endowment and interest rates.) The years 2002–2005, in which Argentina defaulted and was excluded from international markets ([Cruces and Trebesch, 2013](#)), are not included in the estimation. The following OLS estimates are obtained.

$$\hat{\Phi} = \begin{bmatrix} 0.42 & -0.28 \\ 0.32 & 0.93 \end{bmatrix}, \quad \hat{\Omega} = \begin{bmatrix} 0.002 & -0.001 \\ -0.001 & 0.001 \end{bmatrix}, \quad \hat{R} = 1.113.$$

This process is approximated with a Markov chain, setting a grid of 15 equally spaced points for both $\ln(y_t^T)$ and $\ln(\frac{R_t}{R})$, yielding 225 exogenous states. To estimate the transition-probability matrix, I use the method proposed by [Terry and Knotek \(2011\)](#) extending [Tauchen \(1986\)](#).¹²

Finally, to approximate the aggregate dynamics of the economy under the optimal and the full-employment policies, I discretize the endogenous state space (d_t) using 1,001 equally spaced points. To approximate the dynamics under a currency peg, I use 251 equally spaced points for debt (d_t) and 250 equally spaced points for the log of previous period wage (w_{t-1}). The next sections present the results of the quantitative analysis.

3.4.2 Policy Functions

This section analyzes the policy functions under the optimal exchange-rate policy and compares them to those under the two benchmark exchange-rate regimes: full-employment and fixed exchange rate.

Figure [3.1](#) shows decision rules for the nominal devaluation rate, the real exchange rate, unemployment, and next-period debt as a function of the state variables: current debt, tradable endowment, and the external interest rate. In each panel, only one

¹²I am grateful to Stephen J. Terry and Edward S. Knotek II for sharing the code for the Markov-chain approximations of vector autoregressions, which were used in this paper to estimate the transition-probability matrix of the stochastic process.

state variable varies (on the horizontal axis), and the remaining state variables are fixed at their unconditional means (under the optimal policy, if the state is endogenous). In each panel, a shaded region depicts the state-space in which the collateral constraint binds under the optimal policy. The panels on the right do not have a shaded region since varying the interest rate – while keeping the rest of the states fixed at their respective means – is not sufficient to make the collateral constraint bind.

The decision rules for the nominal devaluation rate and real exchange rate under the optimal policy are nonmonotonic, in sharp contrast with the decision rules under the full-employment or fixed exchange-rate policies. The change of the sign in the slope under the optimal policy occurs at the point at which a higher initial level of debt or a lower tradable endowment entails a binding credit constraint. In the region of nonbinding collateral constraint, the decision rules of optimal and full-employment policies coincide, as implied by Proposition 3. In this region, currency depreciation is increasing in the initial debt level and the interest rate, and decreasing in the level of tradable endowment. In the region of binding collateral constraint, while currency depreciation in the full-employment policy continues to be increasing in the initial debt level and decreasing in the level of tradable endowment, currency depreciation under the optimal exchange-rate policy becomes decreasing in the initial debt level and increasing in the level of tradable endowment. Positive unemployment emerges under the optimal exchange-rate policy in the region of binding collateral constraint, increasing in the initial level of debt and decreasing in the level of tradable endowment.

The decision rule for next-period debt under the optimal policy is monotonic, as it would be without an endogenous collateral constraint. Again in sharp contrast, under the full-employment policy, the decision rule for next-period debt is nonmonotonic, with a change in the slope at the point at which a higher initial level of debt or a lower tradable endowment implies a binding credit constraint (for a similar result in

an endowment economy, see [Bianchi, 2011](#)). Hence, consistent with Proposition 4, the optimal policy restores the monotonicity in the policy functions of debt by making the decision rule of the real exchange rate nonmonotonic. In other words, the optimal choice under the credit-access–unemployment trade-off implies no corner solution: the optimal policy is willing to choose unemployment in the region of binding collateral constraint to allow for a higher next-period debt.

The decision rules for the currency peg show that this regime, in contrast to the optimal and full-employment policies, implies positive unemployment in the state-space regions in which the collateral constraint does not bind. In these regions, the fixed exchange-rate regime makes the downward rate rigidity binding. Consistent with this, the decision rule of the real exchange rate under the currency peg displays less sensitivity than that of the other two exchange-rate regimes.

3.4.3 Optimal Exchange-Rate Policy during Financial Crises

Under no binding collateral constraints, the optimal exchange-rate policy always consists of depreciating the nominal exchange rate in response to negative shocks to achieve full employment, as implied by Proposition 3. This section characterizes the optimal exchange-rate policy under periods of binding collateral constraints, or *financial crises*, and compares the dynamics of the economy under the different exchange-rate regimes.

To do this, the calibrated version of the model is simulated for 2 million quarters, identifying periods in which the collateral constraint is binding under the optimal exchange-rate policy. The beginning of a financial crisis episode ($t = 0$) is defined as the first period in which the collateral constraint binds. The responses of the variables, during all episodes of financial crisis, are then averaged.

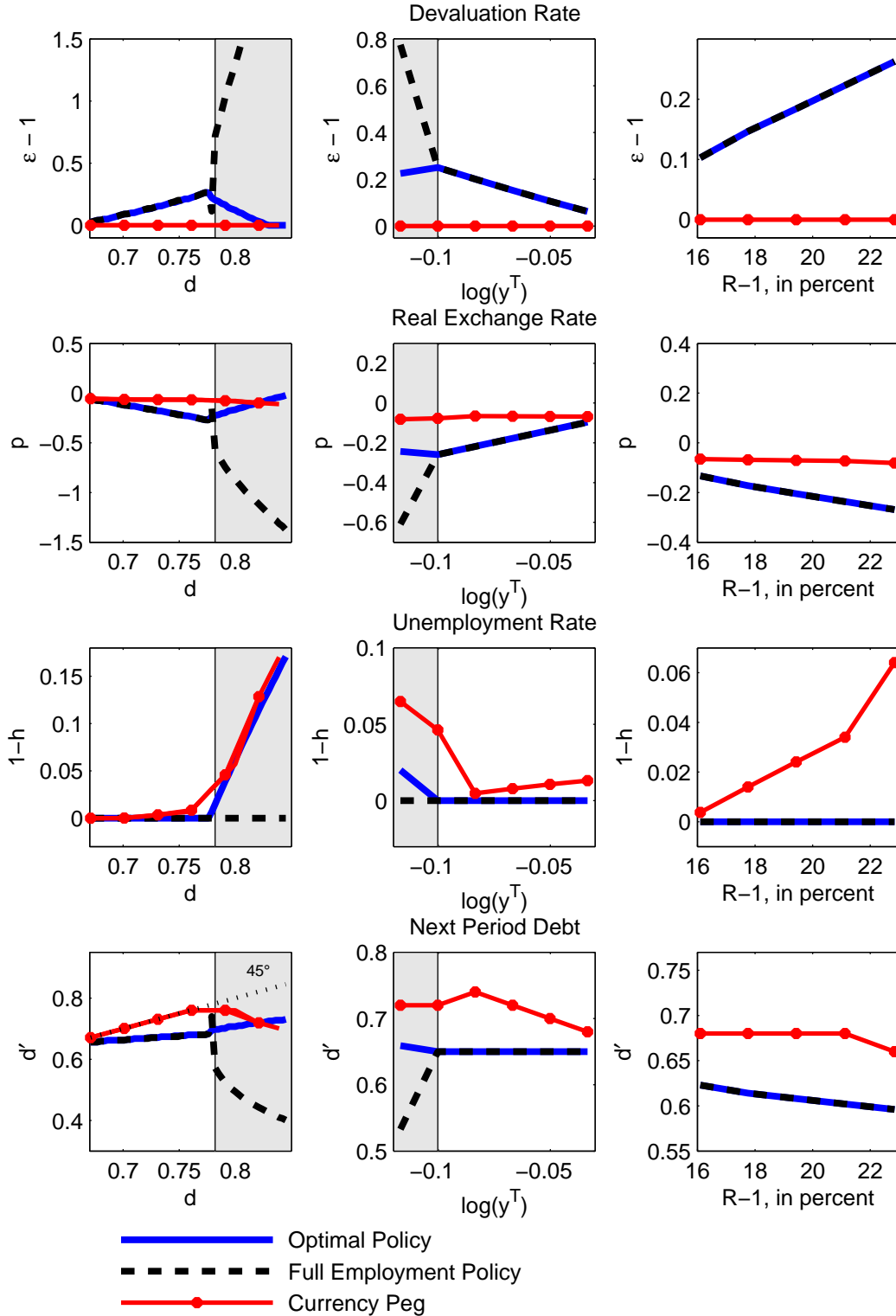


FIGURE 3.1: Policy Functions.

Note: The real exchange rate is expressed in log deviations from its sample mean. Devaluation rate, unemployment rate and next-period debt are expressed in levels. In each panel, only one state variable varies (on the horizontal axis), and the remaining state variables are fixed at their unconditional means (under the optimal policy, if the state is endogenous). Shaded regions denote regions of the state-space in which the collateral constraint binds under the optimal policy.

Figure 3.2 depicts the average external shock during a financial-crisis episode. In the two years that precede such an episode, tradable endowment contracts and interest rates increase. At the crisis trough ($t = 0$), tradable output is 10 percent below its mean, and the annual interest rate is 16 percent, 4 percentage points above its mean. In the three years following the trough, both tradable output and the interest rate recover their precrisis levels.

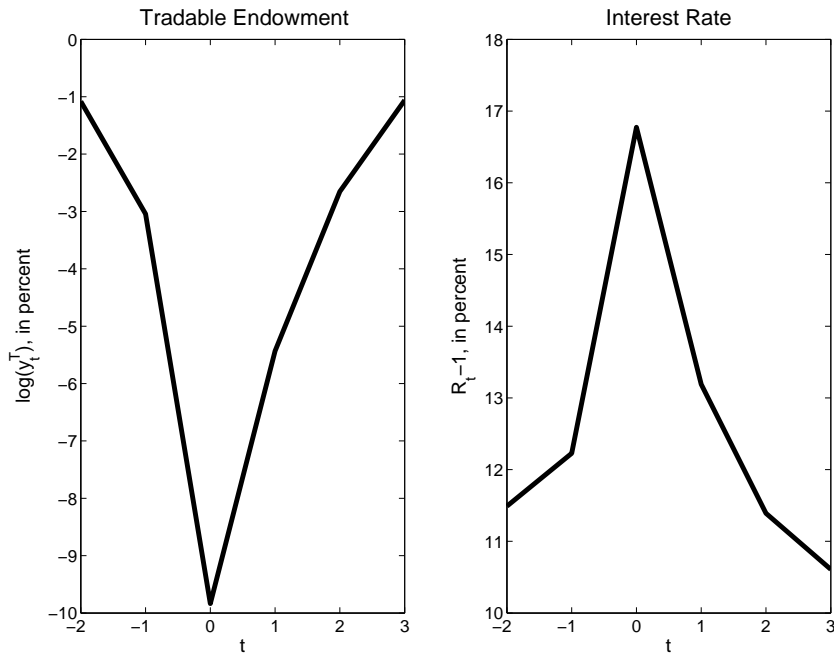


FIGURE 3.2: Financial Crises – Exogenous Variables.

The average responses of the nominal exchange rate and endogenous variables under the different exchange-rate regimes are shown in Figure 3.3. Optimal and full-employment exchange-rate policies display striking similarities and offer a sharp contrast to the response under a currency peg. Even under binding collateral constraints ($t = 0$), the optimal exchange-rate policy does not fix but it substantially depreciates the nominal exchange rate, 52 percent on average. This depreciation is less than that under the full-employment policy (71 percent). As a result, some involuntary

unemployment emerges under binding collateral constraints (1.6 percent on average at the crisis trough). However, unemployment under the optimal exchange-rate policy is significantly lower than that observed under the currency peg (6.2 percent on average at the crisis trough).

In periods of binding collateral constraints, the large real-exchange-rate depreciation under the optimal exchange-rate policy (the relative price of nontradable goods being 39 percent below its mean at the crisis trough), implies a large adjustment of external debt and tradable consumption. Under the optimal policy, the contraction of tradable consumption is much larger than the contraction in nontradable consumption: at the crisis trough, tradable consumption is 20.8 percent below its mean, while nontradable consumption is only 1.2 percent below its mean. The intuition for this result is that while the benefits of reducing unemployment are related to higher nontradable consumption (by market clearing of nontradables) its costs are related to intertemporal misallocation of consumption. In this sense, sudden stops (understood as large current-account adjustments), are in fact part of the endogenous response to large negative external shocks under the optimal exchange-rate policy to prevent greater unemployment. This is again in sharp contrast to the behavior under the currency peg, where, for the same exogenous shock, external debt continues increasing and the current-account deficit expands; at the crisis trough, tradable consumption is 7.6 percent below its mean and nontradable consumption 4.2 percent below its mean.

The large, but still contained, optimal currency depreciation during periods of financial crises is consistent, for instance, with the typical behavior observed in EMs during the global financial turbulence of 2008 (see Figure 3.4). During this episode, EMs considerably depreciate the exchange rate (24 percent on average), but also contain the depreciation, as can be observed in the fall in international reserves. Calvo (2013) shows that this pattern of large nominal depreciation (more than 20 percent on average) with simultaneous exchange-rate intervention is the typical policy observed

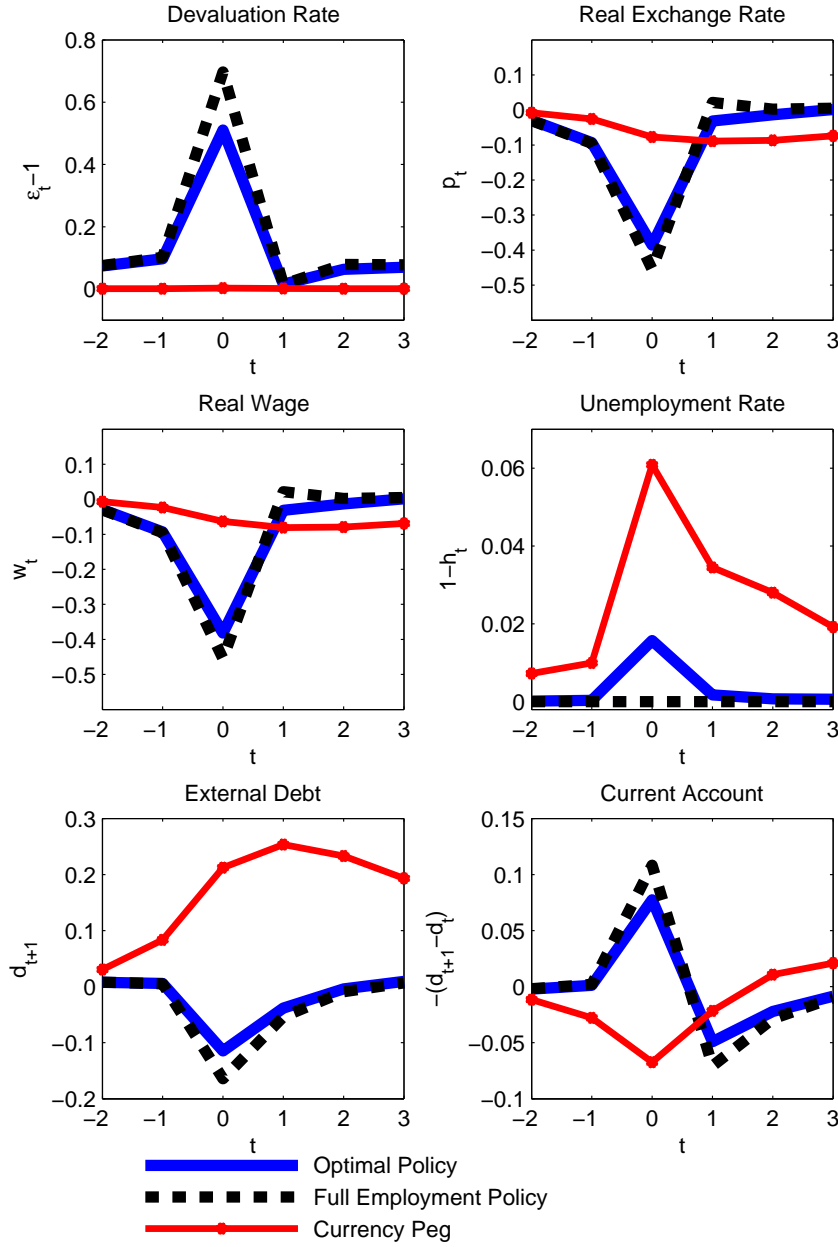


FIGURE 3.3: Financial Crises – Endogenous Variables.

Note: Real exchange rate, real wage, and external debt expressed in log deviations from their sample means. Current account expressed in deviations of its sample mean. Devaluation rate and unemployment rate expressed in levels.

in EMs during periods of sudden stops since 1980.

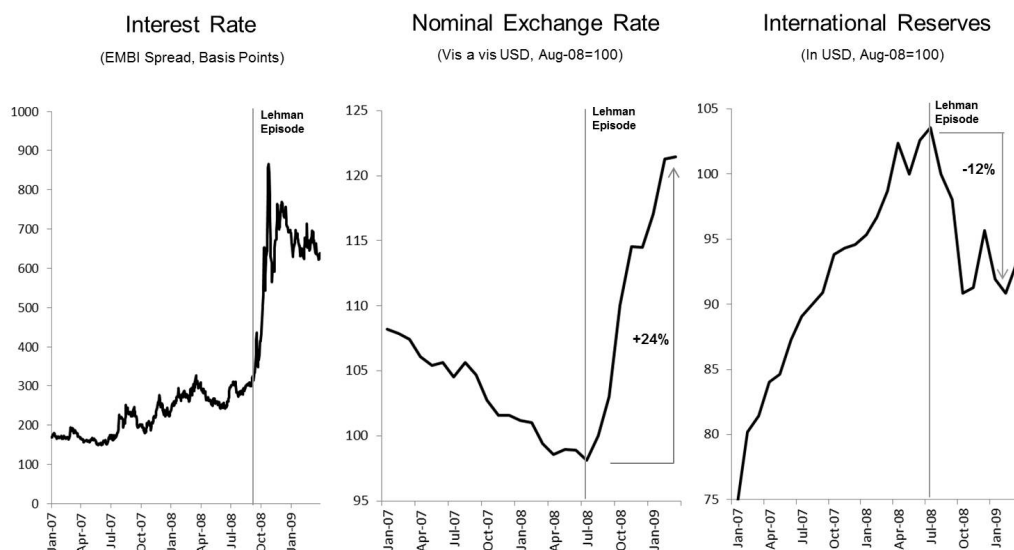


FIGURE 3.4: Exchange-Rate Policy in Emerging Market Economies– Lehman Episode.

Note: Nominal exchange rate and international reserves figures were computed as the simple average for the countries included in the EMBI, except countries with no separate legal tender (Ecuador, El Salvador, and Panama). The countries included in the sample are Algeria, Argentina, Belarus, Belize, Brazil, Bulgaria, Chile, China, Colombia, Cote d'Ivoire, Croatia, Czech Republic, Dominican Republic, Egypt, Gabon, Georgia, Hungary, Indonesia, Jamaica, Jordan, Kazakhstan, Korea, Lebanon, Lithuania, Malaysia, Mexico, Morocco, Namibia, Nigeria, Pakistan, Peru, Philippines, Poland, Romania, Russia, Senegal, South Africa, Sri Lanka, Thailand, Trinidad and Tobago, Tunisia, Turkey, Ukraine, Uruguay, Venezuela, and Vietnam. Data sources: See Data appendix.

3.4.4 Means and Volatilities by Exchange-Rate Regime

Table 3.2 shows that the differences between the optimal and full-employment policies during periods of binding collateral constraint (analyzed in Section 3.4) translate, on the one hand, into a lower volatility of tradable consumption and total consumption, and, on the other hand, into a higher average unemployment rate. This reflects the fact that, under binding collateral constraints, the optimal policy allows for lowering nontradable consumption to improve intertemporal allocation of consumption. The differences in first and second moments between optimal and full-employment policies are slight since the unconditional probability of binding collateral constraints is low (1.5 percent).

The currency peg displays a larger difference in terms of the average unemployment rate with respect to the optimal exchange-rate regime. The reason is that, as

previously analyzed, currency pegs also display unemployment when the collateral constraint does not bind but the wage rigidity does. This response of currency pegs to negative shocks also results in a higher volatility of nontradable consumption and total consumption with respect to the other two regimes.

3.4.5 Welfare and Exchange-Rate Regimes

This section compares welfare under the different exchange-rate regimes. The welfare costs of an exchange-rate regime i with respect to an exchange-rate regime j are computed as the percentage increase in the consumption stream under exchange-rate regime i that will make the representative household indifferent between that consumption stream and the one under the exchange-rate regime j . Formally, the compensation rate under the regime i with respect to regime j , $\lambda^{i,j}$, in a state s_t is implicitly defined by

$$E \left\{ \sum_{k=0}^{\infty} \beta^k U \left(c_{t+k}^i \left(1 + \lambda^{i,j}(s_t) \right) \right) \mid s_t \right\} = E \left\{ \sum_{k=0}^{\infty} \beta^k U \left(c_{t+k}^j \right) \mid s_t \right\},$$

where $i, j \in \{OP, FE, PEG\}$.

Under the assumed form of period utility function, it follows that

$$\lambda^{i,j}(s_t^i) = \left[\frac{V^j(s_t^i)(1-\sigma) + (1-\beta)^{-1}}{V^i(s_t^i)(1-\sigma) + (1-\beta)^{-1}} \right]^{\frac{1}{1-\sigma}} - 1.$$

Since welfare costs are state dependent, Figure 3.5 begins by showing the welfare costs of the full-employment and fixed exchange-rate policies, with respect to the optimal exchange-rate policy, as functions of the states, and the welfare cost of the fixed exchange-rate policy, with respect to the full-employment exchange-rate policy, as a function of the states. As in Figure 3.1, in each panel only one state variable varies (on the horizontal axis), and the remaining state variables are fixed at their unconditional means (under the optimal policy, if the state is endogenous). In each panel, a shaded region show where in the state space the collateral constraint binds

TABLE 3.2
MEANS AND VOLATILITIES BY EXCHANGE RATE REGIME

	OP	FE	CP
Means			
$\mu(c)$	0.98	0.98	0.98
$\mu(c^T)$	0.94	0.94	0.95
$\mu(c^N)$	1.00	1.00	0.99
$\mu(p)$	2.09	2.09	2.16
$\mu(u)$	0.04	0.0	0.8
$\mu(d)$	0.64	0.64	0.52
Volatilities			
$\sigma(c)$	2.86	2.89	3.0
$\sigma(c^T)$	8.3	8.4	5.8
$\sigma(c^N)$	0.4	0	2.3
$\sigma(p)$	40.7	41.3	25.3
$\sigma(u)$	0.5	0.0	2.9
$\sigma(d)$	0.04	0.04	0.12

Note: OP, FE and CP denote optimal exchange-rate policy, full-employment exchange-rate policy, and currency peg, respectively, as defined in Section 3.3. The variables c , c^T , c^N , p , u , and d , denote, respectively, consumption, tradable consumption, nontradable consumption, relative price of nontradables, unemployment rate, and external debt. Volatilities and mean unemployment expressed in percent. Moments computed using parameters from Table 3.1.

under the optimal policy. As in Figure 3.1, the panels on the right do not have a shaded region since varying the interest rate – while keeping the rest of the states fixed at their respective means – is not sufficient to make the collateral constraint bind.

The welfare costs of the full-employment policy with respect to the optimal policy are increasing in the initial debt level, and decreasing in the level of tradable endowment and the interest rate. Welfare costs of the full-employment policy are significantly higher in the regions of the state space in which the collateral constraint binds. The higher welfare costs of the full-employment policy in this region stem from the fact that in these states, the decision rules from the optimal policy differ from those of the full-employment policy, implying a looser credit limit, as shown in the study of the policy functions in Section 3.4. This suggests that the welfare costs

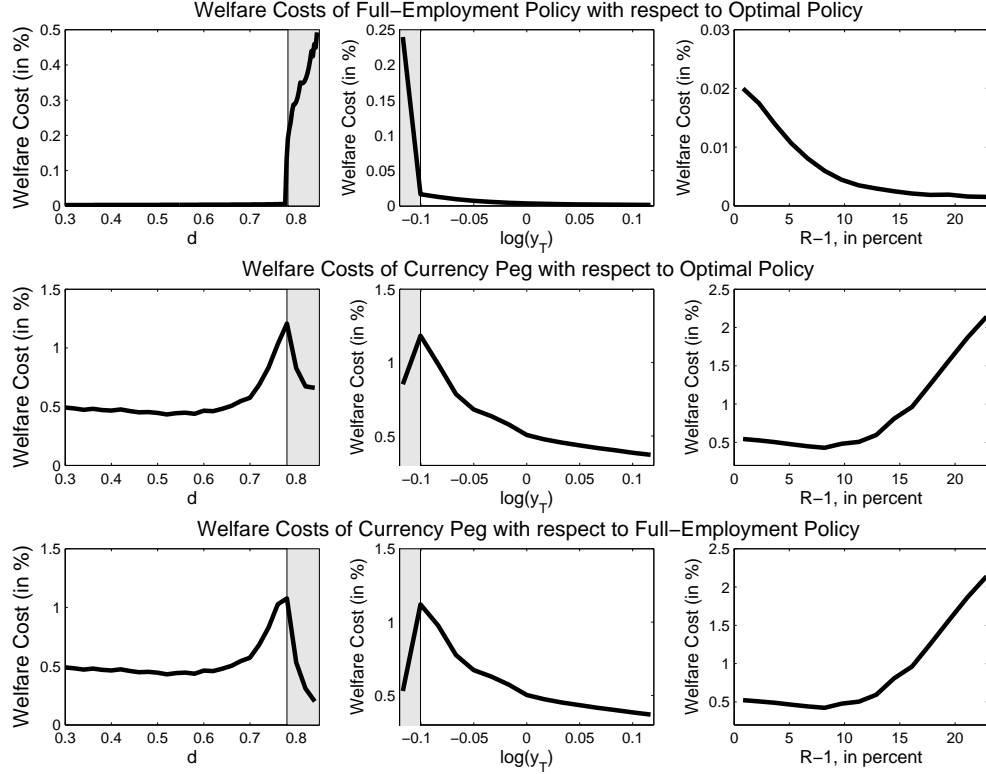


FIGURE 3.5: Welfare Costs by State.

Note: the welfare costs of an exchange-rate regime i with respect to an exchange-rate regime j are defined as the percentage increase in the consumption stream under exchange-rate regime i that will make the representative household indifferent between that consumption stream and that under the exchange-rate regime j in a given state. In each panel, only one state variable varies (on the horizontal axis); the remaining state variables are fixed at their unconditional means (under the optimal policy, if the state is endogenous). Shaded regions denote regions of the state-space in which the collateral constraint binds under the optimal policy.

of the full-employment policy are decreasing in the interest rate because a higher interest rate leads to a reduction in the shadow value from relaxing the constraint.

The welfare costs of the currency peg with respect to the optimal and full-employment policies are nonmonotonic. In the region of nonbinding collateral constraint, welfare costs are decreasing in the level of endowment, and for high levels of debt or interest rates, welfare costs are increasing in the initial debt level and the interest rate. The intuition is that in the region of nonbinding collateral constraint, while the optimal policy maintains full-employment (Proposition 3), the currency peg

TABLE 3.3
WELFARE COSTS BY EXCHANGE-RATE REGIME

Welfare Costs of: with respect to:	Full-Employment Policy Optimal Policy	Currency Peg Optimal Policy	Currency Peg Full-Employment Policy
Mean	0.006	0.576	0.568
Standard Deviation	0.023	0.324	0.325
Maximum	2.7	5.065	5.063
Minimum	0.001	0.22	−1.71

Note: welfare costs expressed in percent. The welfare costs of an exchange-rate regime i with respect to an exchange-rate regime j are defined as the percentage increase in the consumption stream under exchange-rate regime i that will make the representative household indifferent between that consumption stream and the one under the exchange-rate regime j in a given state.

displays a positive level of unemployment, which is increasing in the initial debt level and the interest rate, and decreasing in the tradable endowment (see Section 3.4). In the regions where the collateral constraint binds, the welfare costs of the unemployment generated by the currency peg are increasing in the initial debt level and decreasing in the level of tradable endowment. The intuition is that, in this region, the optimal policy displays a positive unemployment, which, as shown in Section 3.4, is increasing in the debt level and decreasing in tradable endowment.

Table 3.3 shows the moment of the distribution of welfare costs and indicates that the average welfare costs of the full-employment policy with respect to the optimal policy (0.006 percent) are significantly lower than the welfare costs of the currency peg with respect to the optimal policy (0.58 percent).¹³

Finally, Figure 3.6 shows welfare costs during financial crisis episodes (as defined in Section 3.4). It can be observed that financial crises are periods in which the welfare costs of both the full-employment policy and the currency peg increase. The

¹³Formally, the mean of the welfare costs of an exchange-rate regime i with respect to an exchange-rate regime j , denoted $\bar{\lambda}^{i,j}$ is given by

$$\bar{\lambda}^{i,j} = \sum_{s_t} \pi^i(s_t) \lambda^{i,j}(s_t)$$

where $\pi^i(s_t)$ denotes the unconditional probability of state s_t under exchange-rate regime i .

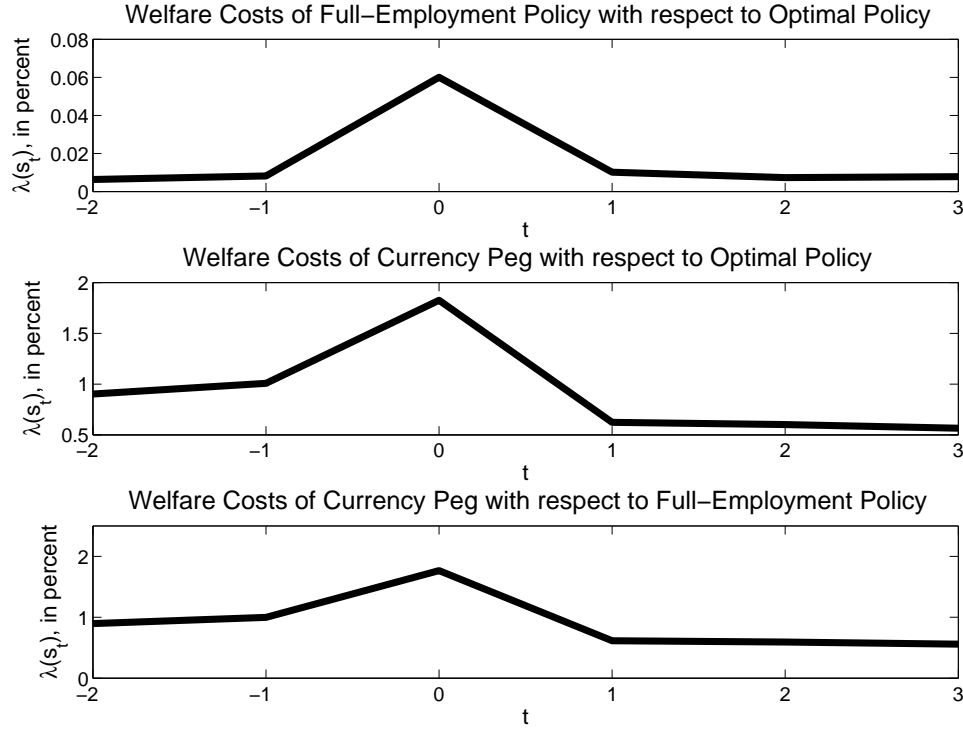


FIGURE 3.6: Welfare Costs During Financial Crises.

Note: The welfare costs of an exchange-rate regime i with respect to an exchange-rate regime j are defined as the percentage increase in the consumption stream under exchange-rate regime i that will make the representative household indifferent between that consumption stream and the one under the exchange-rate regime j in a given state. See Section 3.4 for the definition of a financial crisis episode.

size of the increase in the welfare costs of the full-employment policy are, again, much smaller than the increase in the welfare costs of currency pegs: At the crisis trough the welfare costs of the full-employment and currency peg policies, with respect to the optimal exchange-rate policy, are 0.06 percent and 1.83 percent, respectively. As a consequence, the welfare costs of the currency peg with respect to the full-employment exchange-rate regime also rise during financial crises, reaching 1.77 percent at the crisis trough, meaning that currency pegs are particularly costly in terms of welfare during periods of binding collateral constraints.

3.4.6 Data and Model Predictions

This subsection compares the data with the predictions of the model, during both financial crises and regular business cycles. Although the structure of the model is relatively simple, several features of the data are in line with the predictions of the model, as in previous literature using similar model structures. The predictions of the model are compared with data from Argentina, the economy for which the model economy was calibrated. Figure 3.7 illustrates the fact that Argentina, as did most EMs, alternated between different exchange-rate regimes during the period of study (Ilzetzi, Reinhart, and Rogoff, 2010). For this reason, the predictions of the three exchange-rate regimes are relevant to a comparison of the data.

Figure 3.8 shows that, in most dimensions, the dynamics of the average sudden stop episode in the data is within the predictions of the model during a financial crisis episode (as defined in Section 3.4).¹⁴ The average episode in the data is constructed using three sudden-stop episodes observed in Argentina in 1982, 1989, and 2001 (Eichengreen, Gupta and Mody, 2006; Calvo, Izquierdo and Talvi, 2006). The current-account reversal, the real-exchange-rate depreciation, and the contraction in real wages observed in the average sudden-stop episode in the data are similar in magnitude to the predictions of the model under the optimal exchange-rate policy. The increase of unemployment in the data is also within the predictions of the model, between the unemployment predicted by the currency peg and that predicted under the optimal policy. This can be related to the fact that, as Figure 3.7 indicates, sudden-stop episodes are periods of transition between exchange-rate regimes. A dimension in which the quantitative behavior of the average sudden-stop episode is not in line with the model is in nontradable-output and consumption. Although the

¹⁴Nominal exchange rates and external debt were excluded from the comparison due to hyperinflation episodes and default episodes that occurred in some of these periods (for hyperinflation episodes, see Sargent, Williams, and Zha, 2009; for default episodes, see Cruces and Trebesch, 2013).

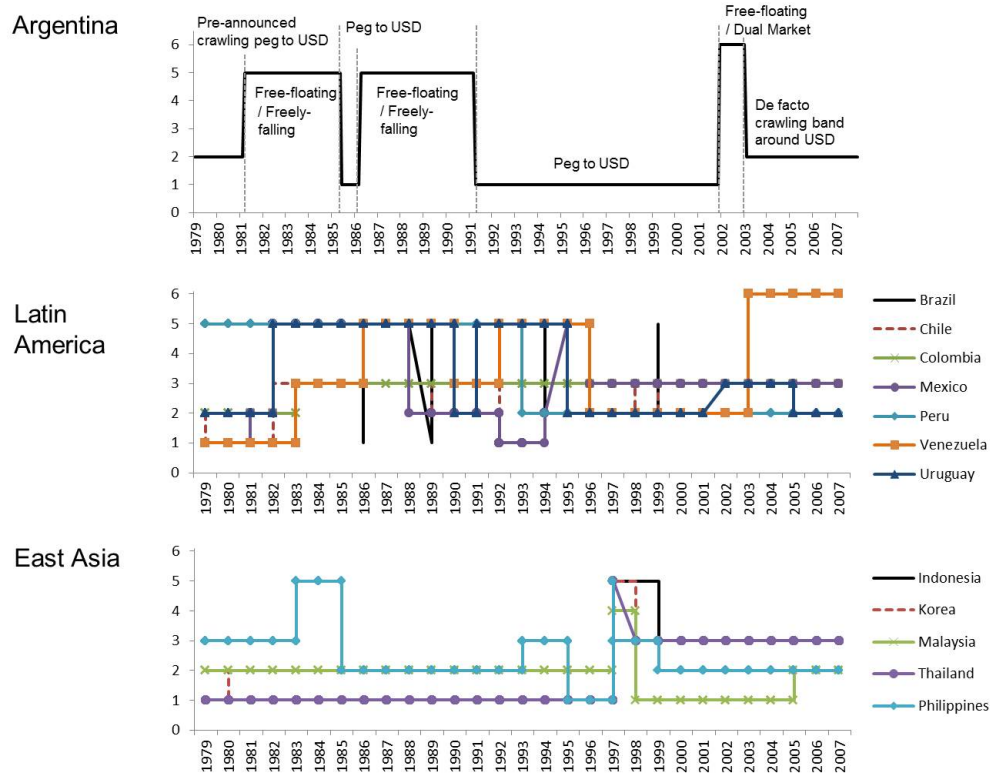


FIGURE 3.7: Exchange-Rate Regimes in Argentina and Emerging Market Economies. *Note:* Data on exchange-rate regimes from [Ilzetzk, Reinhart, and Rogoff \(2010\)](#). Classification codes: 1. No separate legal tender, preannounced peg or currency board arrangement, preannounced horizontal band that is narrower than or equal to ± 2 percent, or de facto peg; 2. preannounced crawling peg, preannounced crawling band that is narrower than or equal to ± 2 percent, de facto crawling peg, or de facto crawling band that is narrower than or equal to ± 2 percent; 3. de facto crawling band that is narrower than or equal to ± 5 percent, moving band that is narrower than or equal to ± 2 percent, managed floating; 4. freely floating; 5. freely falling; 6. dual market in which parallel-market data is missing.

model predicts a significant contraction in these two variables, the contraction observed in the data is larger. Since the behavior of unemployment in the data is in line with the predictions of the model, a key factor driving the larger fall in output in the data is the contraction in measured total factor productivity (TFP), typically observed during sudden stop episodes (see [Calvo et al., 2006](#)). In particular, the average sudden stop episode in the data displays a contraction in measured TFP of 7 percent.¹⁵ However, to maintain a simple structure, the model does not feature

¹⁵Data on measured TFP for Argentina obtained from [Aravena and Fuentes \(2013\)](#). I am grateful to Claudio Aravena for sharing the data on measured TFP.

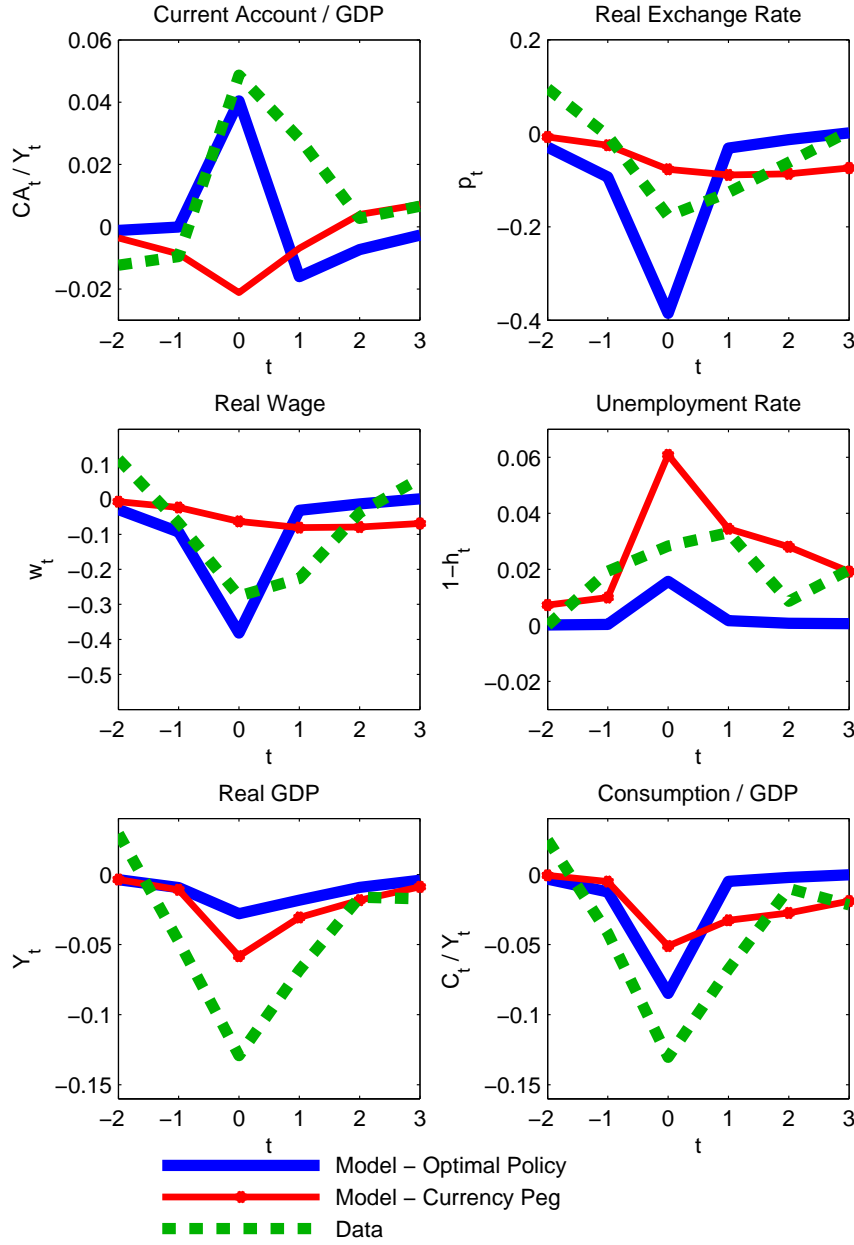


FIGURE 3.8: Financial Crises – Model and Data.

Note: Real exchange rate, real wage, real GDP and consumption expressed in log deviations from their sample means in the model, and from a log quadratic trend in the data. Current-account-to-GDP ratio expressed in deviations from its sample mean in the model and from a quadratic trend in the data. Unemployment rate expressed in levels in the model, and in deviation from $t - 2$ values in the data. See Section 3.4 for the definition of a financial crisis episode. Data sources: see Data appendix.

TABLE 3.4
SECOND MOMENTS – DATA AND MODEL

	Data	Model Predictions		
		OP	FE	CP
Volatilities				
$\sigma(Y)$	8.2	1.6	1.5	2.6
$\sigma(C)/\sigma(Y)$	1.1	1.8	1.9	1.2
$\sigma(u)$	3.2	0.5	0.0	2.9
$\sigma(p)$	19.2	40.7	41.3	25.3
$\sigma(w)$	23.3	30.5	31.0	17.3
$\sigma(TB/Y)$	2.8	2.0	2.1	1.6
$\sigma(CA/Y)$	2.8	1.4	1.5	1.6
Correlations with output				
$\rho(C, Y)$	98.6	75.2	72.8	86.3
$\rho(u, Y)$	-23.0	-28.5	-	-81.1
$\rho(p, Y)$	76.3	77.8	78.8	54.3
$\rho(w, Y)$	78.5	77.7	78.8	51.8
$\rho(TB/Y, Y)$	-95.5	-29.6	-28.6	-6.4
$\rho(CA/Y, Y)$	-88.3	7.9	7.8	41.0
Autocorrelations				
$\rho(Y_t, Y_{t-1})$	74.9	55.3	57.3	49.1
$\rho(C_t, C_{t-1})$	74.9	36.0	33.6	53.0
$\rho(u_t, u_{t-1})$	75.3	8.7	-	27.5
$\rho(p_t, p_{t-1})$	68.1	43.3	41.4	83.8
$\rho(w_t, w_{t-1})$	62.3	43.3	41.4	87.9
$\rho(TB_t/Y_t, TB_{t-1}/Y_{t-1})$	70.2	10.3	5.5	46.7
$\rho(CA_t/Y_t, CA_{t-1}/Y_{t-1})$	56.5	-12.9	-16.7	33.7

Note: Values expressed in percent. OP, FE and CP denote optimal exchange-rate policy, full-employment exchange-rate policy, and currency peg, respectively, as defined in Section 3.3. The variables Y , C , TB , and CA denote, respectively, output, consumption, trade balance, and current account, expressed in real terms; the variables u , p , and w denote, respectively, unemployment rate, relative price of nontradables, wages in units of tradables. Moments in the data were computed for the period 1980–2011. Variables in the data were quadratically detrended. Moments predicted by the model were computed using parameters from Table 3.1. Data sources: see Data appendix.

TFP shocks in the nontradable sector, or an endogenous propagation mechanism that shows up as measured TFP (e.g., capacity utilization, as in Gertler et al., 2007, or imported intermediate inputs as in Mendoza and Yue, 2012).

Table 3.4 shows that, in several dimensions, the second moments observed in the Argentinean data are in line with the second moments predicted by the model. The

observed standard deviations of the unemployment rate, the real wage and the real exchange rate are in line with those predicted by the model. The data is closer to the predictions of the model under a currency peg, which can be related to the fact that, as illustrated in Figure 3.7, in more than 60 percent of the periods since 1980, Argentina was under either a peg or a narrow crawling peg. The model also captures that consumption volatility exceeds output volatility, which is a key feature of EM business cycles (Aguiar and Gopinath, 2007; Uribe and Schmitt-Grohe, 2014). The ratio of standard volatility of consumption to standard volatility of output observed in the data is 1.1, close to the 1.2 predicted under a currency peg. Despite the fact that the volatility of unemployment in the data is similar to the volatility predicted by the model, the volatility of output in the data is significantly higher than the volatility of output predicted by the model. Similar to what was discussed for sudden-stop episodes in the previous paragraph, the higher volatility of output in the data, with respect to the model, can be explained by measured TFP: If the variations in the Solow residual observed in the data are extracted from output, the volatility of output in Argentina in the period of study decreases to 1.9 percent, which is in line with the predictions of the model.

Another relevant dimension observed in the data for Argentina, and in general for EMs, which is qualitatively captured by the model is the countercyclical trade balance (Aguiar and Gopinath, 2007; Uribe and Schmitt-Grohe, 2014). However, the correlation of the trade balance with output observed in the data is less in absolute value than the one predicted by the model. Since the correlation with net factor income from abroad is strongly procyclical both in the data and in the model, the model predicts a procyclical current account. As discussed in Benigno et al. (2012a), an element that could make the model predict a more countercyclical trade balance and current account is the presence of investment (see, for example, Backus, Kehoe, and Kydland, 1993). For the rest of the variables, the correlations with output

observed in the data are in line with those predicted by the model. Finally, the autocorrelations observed in the data are also consistent with those predicted by the model. For output, consumption, unemployment, trade balance and the current account, the autocorrelation coefficient observed in the data are higher than those predicted by the model, while the correlation coefficient of the real wage and the real exchange rate observed in the data are within the model's predictions.

3.5 Sensitivity Analysis and Extensions

This section studies how the characterization of the optimal exchange-rate policy during financial crises is affected by alternative parametrizations of the model, alternative shocks, and alternative modeling assumptions.

3.5.1 Parameter values

This section shows that the main conclusions regarding the characterization of the optimal exchange-rate policy during financial crises are robust to alternative parameter values. In particular, Figure 3.9 shows the average values of nominal exchange-rate depreciation, real exchange rate, unemployment, and tradable consumption under the optimal exchange-rate policy during periods in which the collateral constraint binds for alternative parameter values. The focus is on financial crises since, as shown in Proposition 3, periods of nonbinding collateral constraint are always characterized by full employment under the optimal exchange-rate policy, independent of parameter values.

I begin by studying alternative values for the intratemporal elasticity of substitution, considering values in the range used in the literature, between $\xi = 0.4$ and $\xi = 0.83$ (see Section 3.3.2 for a survey). The value of this parameter used in the baseline calibration is $\xi = 0.44$, following the estimates of [Gonzalez-Rozada et al.](#)

(2004) for Argentina. As explained in Section 3.3.2, this parameter determines the extent to which exchange-rate depreciations decrease collateral values. If $\xi \geq 1$, there is no negative effect of a currency depreciation on collateral values, and the optimal policy is always to achieve full employment. As expected from this, Figure 3.9 indicates that the higher the value of the intratemporal elasticity of substitution, the lower the unemployment rate under the optimal exchange-rate policy. In this sense, the conclusions obtained in the baseline calibration are conservative with respect to this parameter value: A higher intratemporal elasticity of substitution would imply an optimal policy even closer to full employment.

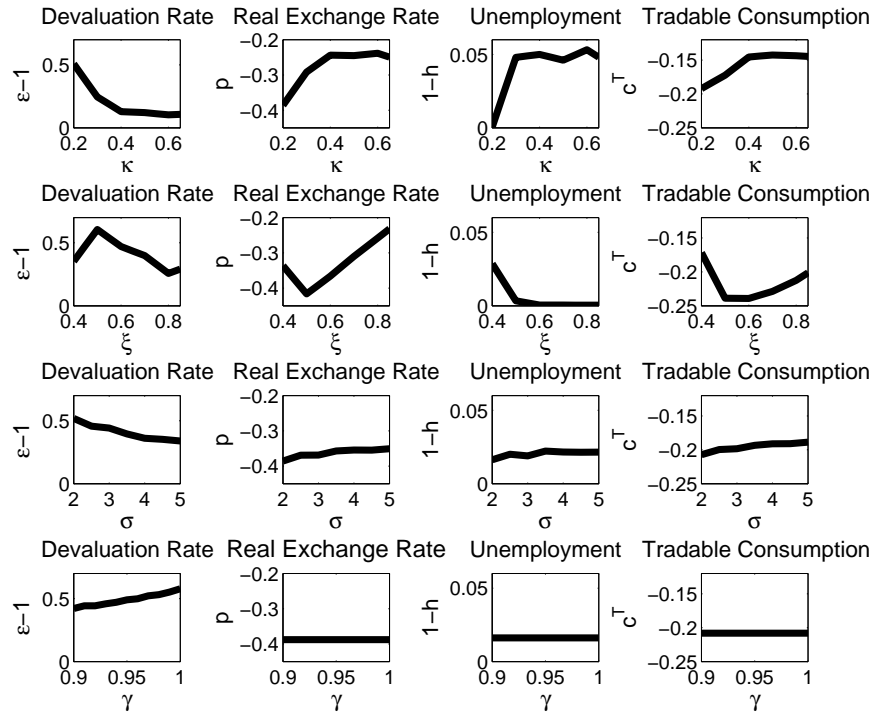


FIGURE 3.9: Sensitivity of Optimal Policy during Financial Crises.

Note: Figures denote the value of each variable at the trough ($t = 0$) of the average financial crisis episode. See Section 3.4 for the definition of a financial crisis episode.

I then study alternative values for κ , the parameter that governs the collateral constraint. To the best of my knowledge, there is no empirical estimate available

of this parameter. In the baseline calibration this parameter was set to $\kappa = 0.263$ to match the probability of sudden stops. I now consider alternative values for the collateral parameter ranging from $\kappa = 0.2$ (average debt-to-GDP ratio in Argentinean data) to $\kappa = 0.645$ (maximum debt-to-GDP ratio in Argentinean data). Results in Figure 3.9 indicate that, in this range of parameter values, the higher the collateral parameter, the lower the depreciation rate under the optimal exchange-rate policy, and the higher resulting unemployment and tradable consumption. The difference is nontrivial: for instance, with a value of $\kappa = 0.645$ the average depreciation rate in a period of financial crisis is 10.3 percent and the resulting unemployment rate, 5.3 percent (which compares to 1.6 percent in the baseline scenario). The intuition for this result is that, the higher the collateral parameter, the higher the effect that containing real exchange-rate depreciation has on collateral values, and thus the higher the benefits of containing depreciation. However, even in this case, it can be observed that the optimal policy features a large real exchange-rate depreciation during financial crises (a 24 percent fall in the relative price of nontradable goods), implying that it is optimal to contract tradable consumption more than nontradable consumption and employment.

In the third place, I study the sensitivity, of the optimal policy during financial crises, with respect to the intertemporal elasticity of substitution. In the baseline calibration, the inverse of the intertemporal elasticity of substitution was set to $\sigma = 2$, a standard value in the business-cycle literature. Since, as studied in Section 3.3.2, the benefits of credit-market access are related to the intertemporal allocation of consumption, this is a key parameter in the optimal exchange-rate policy. The range of values of the inverse of the intertemporal elasticity of substitution considered is from $\sigma = 2$ to $\sigma = 5$, the value estimated in Reinhart and Vegh (1995) for Argentina. Results for this range of parameter values are shown in Figure 3.9. As expected, a lower intertemporal elasticity of substitution is associated with a lower optimal

exchange-rate depreciation during financial crises. Quantitatively, the conclusions are similar to those obtained in the baseline calibration. For instance, for $\sigma = 5$ the optimal policy in financial crises is a large nominal and real exchange-rate depreciation of 35 percent.

Finally, it is worth mentioning that γ , the degree of wage rigidity, does not affect the allocation under the optimal exchange-rate policy, except for the value of the optimal nominal depreciation rate, ϵ_t . This can be seen from the fact that γ does not enter in the formulation of the Bellman equation (3.19) that describes the dynamics of $\{d_{t+1}, c_t^T, h_t\}$ under the optimal exchange-rate policy. It follows that the real exchange-rate depreciation, under the optimal exchange-rate policy, does not depend on γ either. The only variable under the optimal exchange-rate policy that is affected by γ is the nominal exchange-rate depreciation: the lower the γ , the lower the average nominal exchange-rate depreciation required to implement the optimal allocation during a financial crisis episode, as illustrated in Figure 3.9.

In summary, the main findings regarding the optimal exchange-rate policy during financial crises in previous sections are robust with respect to alternative values of structural parameters: The optimal exchange-rate policy implies, on average, large real exchange-rate depreciations during financial crises; this is achieved by depreciating the nominal exchange rate, and implies a relatively small increase in the unemployment rate as compared to the decline in tradable consumption.

3.5.2 Stochastic structure

The baseline quantitative analysis (Section 3.4) includes interest rate shocks, and uses the contractual interest rate to calibrate these shocks. This section considers instead three alternative stochastic structures: i) an economy with interest-rate shocks calibrated to those of the risk-free rate; ii) an economy with no interest-rate shocks; and iii) an economy with no interest rate shocks but shocks to the parameter κ , that

governs the collateral constraint. These sensitivity analyses are important because, in the presence of credit constraints, calibrating the model using the EM's contractual interest rate (as in the baseline calibration), which incorporates default risk, could lead to overestimating the costs of borrowing during crises. It is shown that the optimal policy under these alternative stochastic structures still features large nominal and real exchange-rate depreciations during financial crises (between 20 and 25 percent). However, since higher interest rates lead to a lower shadow value from relaxing the credit constraint, the optimal currency depreciation under these stochastic structures is half of that under the baseline stochastic structure.

The first case considers an economy with interest-rate shocks calibrated to those of the risk-free rate. As in the baseline calibration, it is assumed that the two exogenous driving forces – tradable endowment and interest rate – follow a first-order VAR of the form in (3.27), with the risk-free rate taking the place of the country's interest rate. The risk-free rate is measured by a US real interest rate (Treasury-bill rate, deflated with a measure of expected dollar inflation, constructed as detailed in the Data appendix). Following [Uribe and Yue \(2006\)](#), I assume that the risk-free rate follows a univariate process (i.e., $\Phi_{21} = 0$). The parameters of the stochastic process are estimated for the same period and using the same methodology as in the baseline calibration. The parameters $\{\beta, a, \kappa\}$ are used, as in the baseline calibration, to match the average level of external debt-to-GDP ratio, the share of tradable output in GDP, and the frequency of sudden stops observed in the data. All the rest of the parameters are set as in the baseline calibration (Table 3.1).

The second case considers an economy with no interest-rate shocks. In this economy, tradable endowment shocks are the only source of uncertainty and are assumed to follow the same stochastic process as in the baseline calibration. The fixed gross interest rate is set to 1.018, the average US real interest rate from 1980 to 2011. Again, as in the baseline calibration, the parameters $\{\beta, a, \kappa\}$ are used to match the average

level of external debt-to-GDP ratio, the share of tradable output in GDP, and the frequency of sudden stops observed in the data, with all the rest of the parameters set as in the baseline calibration (Table 3.1).

Finally, the third case considers an economy with no interest-rate shocks, but instead with shocks to the parameter κ , that governs the collateral constraint. These types of shocks have been used in the literature to capture sudden stops driven by shocks to foreign investors' confidence in EMs (see, for example, Benigno and Fornaro, 2012; Bianchi, Hatchondo and Martinez, 2013). Formally, the collateral constraint (3.18) is now replaced with

$$d_{t+1} \leq \kappa_t \left(y_t^T + \left(\frac{1-a}{a} \right) (c_t^T)^{\frac{1}{\xi}} F(h_t)^{\frac{\xi-1}{\xi}} \right), \quad (3.28)$$

where κ_t follows a first-order Markov process. As is standard in this literature, we assume for simplicity that $\kappa_t \in \{\kappa^L, \kappa^H\}$, with $\kappa^L < \kappa^H$. The value of κ^H is set to an arbitrarily high value such that the collateral constraint never binds in period t if $\kappa_t = \kappa^H$. Similar to the baseline calibration, the parameters $\{\beta, a, \kappa^L\}$ are used to match the average external debt-to-GDP ratio, the share of tradable output in GDP, and the frequency of sudden stops observed in the data. Following Benigno and Fornaro (2012), the probability of entering a low-collateral-constraint state, denoted π_κ^L , is set to 0.1 (Jeanne and Ranciere, 2011), and the probability of exiting a low-collateral-constraint state, denoted π_κ^H , is set to 0.5 (Alfaro and Kanczuk, 2009). The other parameters are set as in the baseline calibration (Table 3.1).

Figure 3.10 displays the optimal nominal exchange rate and endogenous variables for the average financial crisis episode (as defined in section 3.4), under the baseline and the three alternative stochastic structures. The optimal nominal and real exchange-rate depreciation during the average financial crisis episode, under the three alternative stochastic structures, is between 20 and 25 percent. Thus, independently of the assumption on the behavior of interest rates during financial crisis episodes, it is optimal to allow for large currency depreciation, and to induce a relatively small

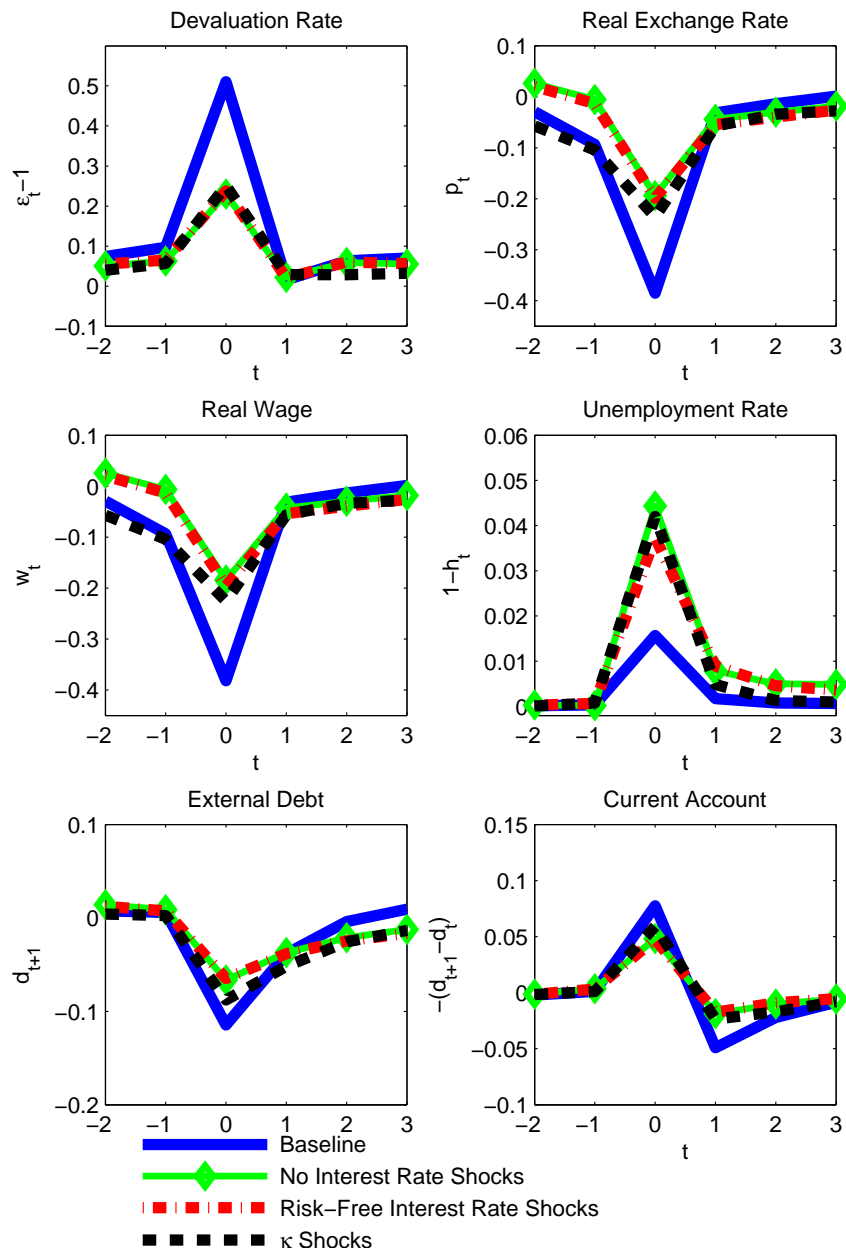


FIGURE 3.10: Financial Crises – Optimal Policy under Alternative Stochastic Structures.

Note: Real exchange rate, real wage, and external debt expressed in log deviations from their sample means. Current account expressed in deviations from its sample mean. Devaluation rate and unemployment rate expressed in levels. See Section 3.4 for the definition of a financial crisis episode.

increase in the unemployment rate compared to the adjustment of tradable consumption. However, the fact that, under the alternative stochastic structures, currency depreciation is half that under the baseline structure suggests that the behavior of interest rates during a financial crisis is key to determining the optimal degree of “fear of floating.” In particular, when a financial crisis occurs with no significant increase in interest rates, optimal policy calls for a more contained depreciation, a smaller current-account adjustment and a larger increase in unemployment than when the episode occurs with a sizable increase of interest rates.

3.5.3 Model structure

This section studies the characterization of optimal exchange-rate policy after relaxing two assumptions of the baseline model: endowment in the tradable sector and inelastic labor supply.

3.5.3.1 Production in the Tradable Sector

This subsection relaxes the baseline model’s assumption of tradable endowment and instead considers production in the tradable sector. This a relevant modification to study since, as shown in [Benigno et al. \(2012a\)](#), labor reallocation is an important mechanism in dealing with financial crises in the presence of a collateral constraint like the one studied in this paper. To facilitate the reallocation mechanism it will be assumed – in sharp contrast to the baseline model – that labor is perfectly mobile across sectors.

As in the nontradable sector, production in the tradable sector is now assumed to be conducted by firms that operate in competitive labor and product markets, each period hiring labor to produce the tradable good, y_t^T , and using an isoelastic production function. Profits each period, expressed in units of tradable goods, are

given by

$$\Pi_t^T = Z_t^T (h_t^T)^{\alpha^T} - w_t h_t^T,$$

where h_t^T denotes labor employed in the tradable sector, Z_t^T denotes productivity in the tradable sector, assumed to be exogenous and stochastic.

The firms' problem is to choose h_t^T to maximize profits given prices w_t and productivity Z_t^T . The first-order condition of this problem is

$$Z_t^T \alpha^T (h_t^T)^{\alpha^T - 1} = w_t. \quad (3.29)$$

Total hours worked is now given by the sum of hours worked in the tradable sector and hours worked in the nontradable sector (denoted h_t^N):

$$h_t = h_t^T + h_t^N. \quad (3.30)$$

The rest of the equilibrium conditions are the same as in the baseline economy. To calibrate the model, the labor share in the tradable sector is set to $\alpha^T = 0.5$, following evidence in [Uribe \(1997\)](#). The parameters $\{\beta, a, \kappa\}$ are used, as in the baseline calibration, to match the average level of external debt-to-GDP ratio, the share of tradable output in GDP, and the frequency of sudden stops observed in the data. All the rest of the parameters are set as in the baseline calibration (Table 3.1). Given the lack of historical sectoral data on measured TFP for Argentina, and to facilitate comparison with the baseline calibration, it is assumed that the stochastic process is the same as in (3.27), with Z_t^T taking the place of y_t^T .

Figure 3.12 shows that the optimal exchange-rate policy in the economy with production in the tradable sector features less unemployment than in the baseline economy. This is because, in this modified setup, currency depreciations have a larger benefit than in the baseline economy, to reallocate resources and increase production in the tradable sector. This increase in tradable production, in turn, reduces the tightening in the collateral constraint and the required current-account adjustment.

Therefore, in equilibrium, even if currency depreciation is smaller than in the baseline economy, unemployment is also smaller than that observed in the baseline economy. These results indicate that the degree of labor mobility across sectors is a key characteristic of the economy for determining the optimal degree of “fear of floating,” and also the severity of financial crises in terms of unemployment and current-account adjustment.

3.5.3.2 Endogenous Labor Supply

In this section, the assumption of inelastic labor supply is relaxed. I assume, instead of (3.1), that households’ preferences are given by the expected utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (U(c_t) - v(h_t)), \quad (3.31)$$

where the function $v(\cdot)$ is assumed to be continuous, twice differentiable, strictly increasing and convex.

The first-order condition with respect to hours worked is given by

$$v'(h_t^s) = w_t \lambda_t, \quad (3.32)$$

where h_t^s denotes the number of hours supplied by households to the labor market.

Actual hours worked cannot exceed labor supply, meaning that labor-market conditions (3.12) and (3.13) are replaced respectively by

$$h_t \leq h_t^s, \quad (3.33)$$

$$\left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) (h_t^s - h_t) = 0. \quad (3.34)$$

The rest of the equilibrium conditions are the same as in the baseline economy.

To calibrate the model, we assume the functional form

$$v(h^s) = -\varphi \frac{(\bar{h} - h_t)^{1-\theta} - 1}{1-\theta}, \quad (3.35)$$

where $\varphi > 0$ and $\theta > 0$. The values of \bar{h} and φ , are set to 3 and 1.5 to match an average level of hours worked at unity (to preserve the size of the nontradable sector), assuming that households at full employment spend one third of their time working. The value of θ is set to 1.6 which corresponds to an elasticity of labor supply of 1.25, following the estimates in [Mendoza \(2010\)](#). The parameters $\{\beta, a, \kappa\}$ are used, as in the baseline calibration, to match the average level of external debt-to-GDP ratio, the share of tradable output in GDP, and the frequency of sudden stops observed in the data. The other parameters are set as in the baseline calibration (Table [3.1](#)).

Figure [3.12](#) shows that the optimal exchange-rate policy in an economy with an endogenous labor supply features significantly more unemployment during financial crises than in the baseline economy, showing that labor elasticity is also a relevant characteristic of the economy for determining the optimal degree of “fear of floating.” The contraction in the level of employment is greater than in the baseline economy since, in this setup, decreasing employment has a benefit not only in terms of relaxing the credit constraint, but also in terms of increasing leisure. However, most of this increase in unemployment is driven by an increase in the labor supply: Unlike the Ramsey planner, agents do not incorporate the effect that increasing employment has on tightening the collateral constraint, and increase labor supply during financial crises, given the large contraction in consumption that occurs during these episodes. While currency depreciation is smaller than in the baseline economy, the optimal allocation is still characterized by a relatively large real exchange-rate depreciation during financial crises (a 21 percent fall in the relative price of nontradable goods), implying a relatively small decrease in nontradable consumption rate compared to that of tradable consumption.

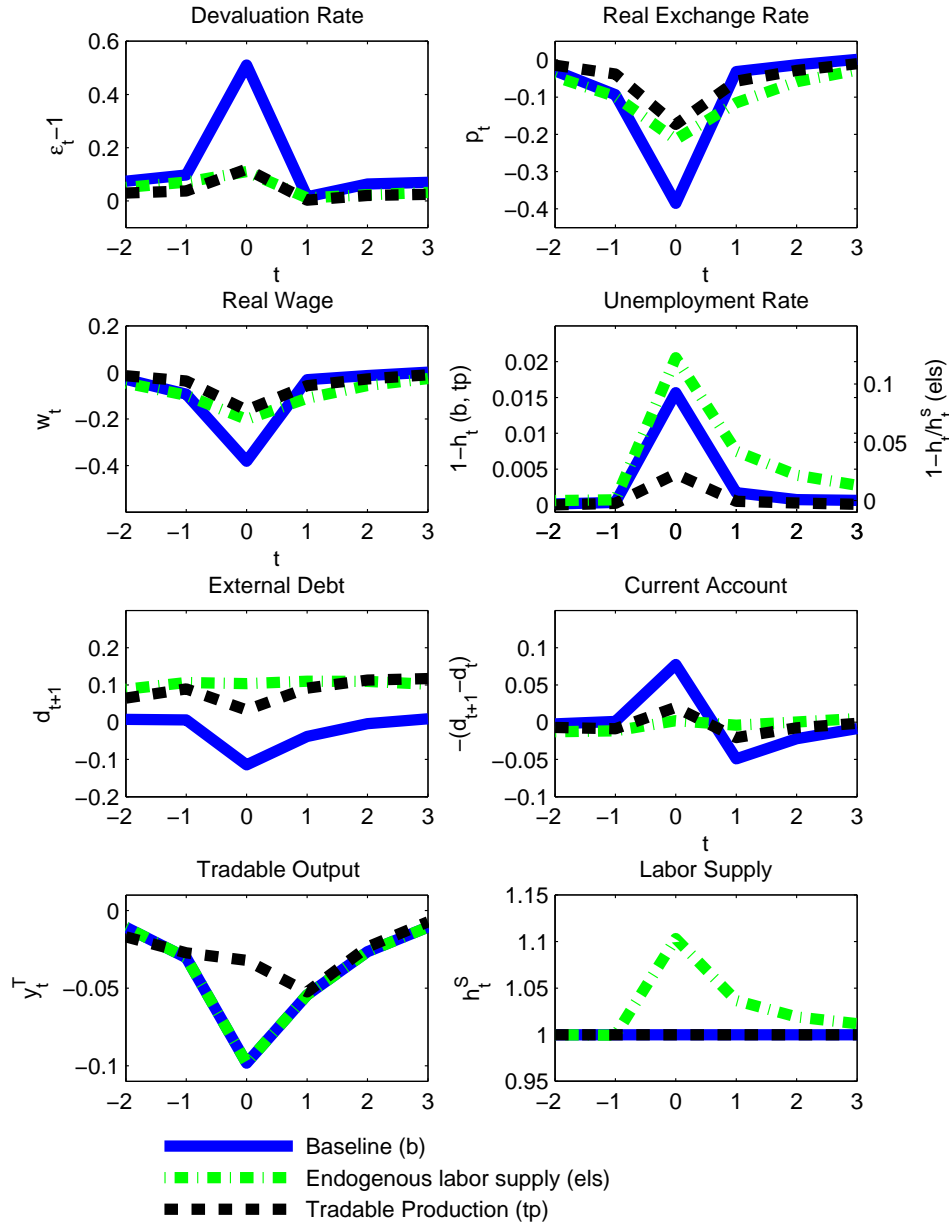


FIGURE 3.11: Financial Crises – Optimal Policy under Alternative Model Structures. *Note:* Real exchange rate, real wage, external debt, and tradable output expressed in log deviations from their sample means. Current account expressed in deviations from its sample mean. Devaluation rate, unemployment rate, and labor supply expressed in levels. See Section 3.4 for the definition of a financial crisis episode.

3.5.4 No Capital-Control Taxes

So far, the paper has characterized the optimal exchange-rate policy as conditional on the use of an optimal capital-control tax. As discussed in Section 3.3, this optimal capital-control tax eliminates any inefficient borrowing that might stem from the collateral constraint or from the downward wage rigidity. This section studies the sensitivity of the optimal policy to this assumption by extending the analysis to the case where the only instrument available to the Ramsey planner is the nominal exchange rate.

The Ramsey planner is assumed to have access to a commitment technology. Since the household's intertemporal optimality condition (3.6) is part of the set of restrictions, the Ramsey problem cannot, as in Section 3.3, be expressed with a standard recursive formulation.¹⁶ However, the method developed in [Marcet and Marimon \(2011\)](#) can be applied to reformulate the nonrecursive problem with forward-looking variables as a recursive saddle-point problem.¹⁷ This approach implies the inclusion of the Lagrange multipliers associated with forward-looking constraints (in this case, equation (3.6)) as costate variables. As shown in Appendix C.4, using this method the dynamics under the optimal exchange-rate policy (without capital-control

¹⁶This problem does not arise when the Ramsey planner has access to an optimal capital-control tax since, as shown in Appendix C.1, the capital-control tax can always be picked so that the optimal allocation satisfies the intertemporal optimality condition (3.6).

¹⁷For related applications of the [Marcet and Marimon \(2011\)](#) method, see [Adam and Billi \(2005\)](#), [Monacelli \(2008\)](#), and [Svenson \(2010\)](#).

taxes) can be expressed with the recursive saddle-point problem:

$$W^{OP} \left(s^X, d, \tilde{\phi}^D \right) = \min_{\tilde{\phi}^{D'}} \max_{d', c^T, h, \mu} \left[H \left(c^T, h, \mu, \tilde{\phi}^D, \tilde{\phi}^{D'} \right) + \beta \mathbb{E}_{s^{X'}} W^{OP} \left(s^{X'}, d', \tilde{\phi}^{D'} \right) \right] \quad (3.36)$$

$$\begin{aligned} s.t. \quad & \frac{d'}{R} = d + c^T - y^T, \\ & d' \leq \kappa \left(y^T + \left(\frac{1-a}{a} \right) (c^T)^{\frac{1}{\xi}} F(h)^{\frac{\xi-1}{\xi}} \right), \\ & d' \leq d^N, \\ & h \leq \bar{h}, \\ & \mu \geq 0, \\ & \mu \left(\kappa \left(y^T + \left(\frac{1-a}{a} \right) (c^T)^{\frac{1}{\xi}} F(h)^{\frac{\xi-1}{\xi}} \right) - d' \right) = 0, \end{aligned}$$

with

$$\begin{aligned} H \left(c^T, h, \mu, \tilde{\phi}^D, \tilde{\phi}^{D'} \right) \equiv & U \left(A \left(c^T, F(h) \right) \right) - \\ & - U_c \left(c^T, F(h) \right) A_T \left(c^T, F(h) \right) \left(\frac{\tilde{\phi}^{D'}}{R} - \tilde{\phi}^D \right) + \tilde{\phi}^{D'} \mu, \end{aligned}$$

where the costate variable $\tilde{\phi}^D$ denotes the Lagrange multiplier of the household's intertemporal optimality condition (3.6) chosen in the previous period by the Ramsey planner and can be interpreted as the value to the planner of promises that must be honored from past commitments. To approximate the dynamics under the optimal exchange-rate policy, the functional equation (3.36) is solved numerically.¹⁸ The parameters used are the same as in Table 3.1.

Figure ?? shows that the dynamics during financial crisis episodes under the optimal exchange-rate policy without capital-control taxes are similar to those under the

¹⁸The presence of additional states and controls in the case of optimal policy without an optimal capital-control tax makes the numerical approximation computationally more demanding. For this reason, the approximation uses a grid of five equally spaced points for both $\ln(y_t^T)$ and $\ln(\frac{R_t}{R})$, 51 equally spaced points for (d_t) , and 21 equally spaced points for $(\tilde{\phi}_t^D)$. For comparison purposes (in this section only) these grids were also used to solve numerically for the optimal exchange rate with optimal capital-control taxes

optimal exchange-rate policy with optimal capital-control taxes. The main difference is that, in the case without capital-control taxes, the optimal allocation displays a higher nominal and real exchange-rate depreciation and, as a result, a smaller unemployment and a larger current-account adjustment. Therefore, without capital-control taxes, the optimal policy commits to make credit-access tighter during financial crisis episodes, increasing the private costs of borrowing.

3.6 Conclusions

This paper conducts a quantitative study of the optimal exchange-rate policy facing a trade-off between credit access and unemployment, which captures a central discussion of the policy debate typically observed during financial crises in EMs. In the presence of downward nominal wage rigidity, allowing for nominal exchange-rate depreciations can help attenuate unemployment. In the presence of liability dollarization and collateral constraints linked to tradable and nontradable income, fighting real exchange-rate depreciation alleviates the consumption adjustment.

The main finding is that the optimal exchange-rate policy during financial crises is consistent with managed-floating exchange-rate regimes, widely used by EMs in periods of sudden stops: It is optimal to allow for large currency depreciation (between a 17 and 40 percent average fall in the relative price of nontradable goods), but also to contain currency depreciation with respect to full-employment levels. The bias of the optimal policy towards large currency depreciation is related to the fact that the welfare costs from unemployment and lower consumption typically outweigh those of intertemporal misallocation of consumption.

The findings of the paper suggest that simple policy recipes will, in general, not be optimal during EMs' financial crises. For instance, both full-employment and fixed-exchange-rate regimes entail relatively large welfare costs compared to the optimal

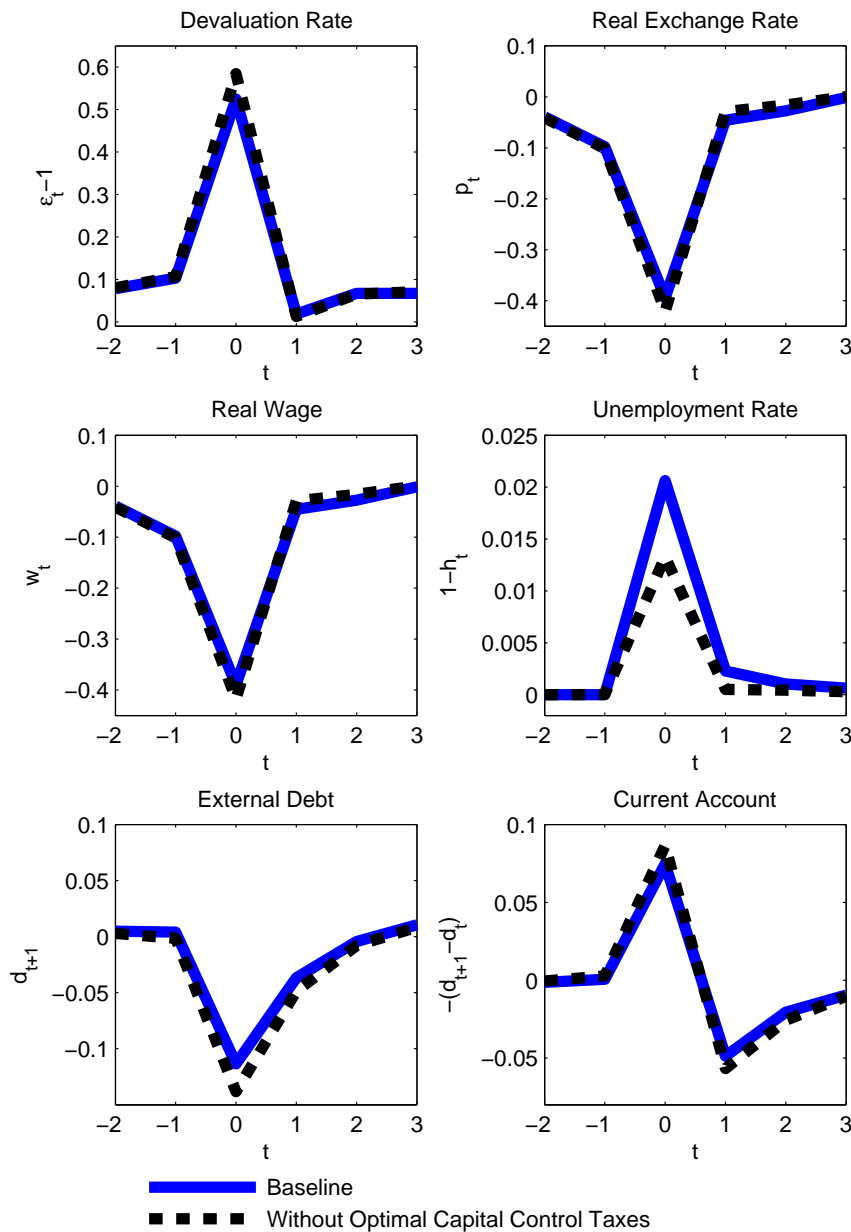


FIGURE 3.12: Financial Crises – Optimal Policy with No Capital-Control Taxes.
Note: Real exchange rate, real wage, and external debt expressed in log deviations from their sample means. Current account expressed in deviations of its sample mean. Devaluation rate and unemployment rate expressed in levels. See Section 3.4 for the definition of a financial crisis episode.

exchange-rate policy during financial crisis episodes – 0.06 percent and 1.8 percent of consumption per period, respectively. Moreover, the paper shows that the optimal

degree of “fear of floating” during financial crises depends on the nature of the shocks, and structural characteristics of the economy. This means that what is optimal during a financial crisis episode in a given economy might not be optimal in another economy or in an episode involving a different combination of shocks.

A novel finding is that sudden stops, understood as large current-account adjustments, are generally part of the endogenous response to large negative shocks under the optimal exchange-rate policy. In other words, while exchange-rate policy could prevent sudden stops by resisting real exchange-rate depreciation, the associated unemployment costs make this policy suboptimal.

In future research, several extensions related to the present paper’s framework could be considered. First, the paper abstracts from capital accumulation. Including capital accumulation would be computationally demanding, but would enrich the study of the trade-off faced by the policymaker. Second, the paper studies the optimal exchange-rate policy when the policymaker has access to a commitment technology. An interesting area of future research is the optimal time-consistent exchange-rate policy in a framework in which the policymaker does not have commitment. Third, the paper assumes that all debt is denominated in a foreign currency. A relevant extension would be a study of the interaction between exchange rate policy and optimal currency composition of external debt under the framework of this paper. These extensions are planned for future research.

Bibliography

- Acemoglu, Daron 2001. "Credit Market Imperfections and Persistent Unemployment." *European Economic Review*, 45(4–6): 665–679.
- Adam, Klaus, and Roberto M. Billi. 2005. "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates," Federal Reserve Bank of Kansas City, RWP 05-07.
- Aghion, Phillipe, Phillipe Bacchetta, and Abhijit Banerjee. 2001. "Currency Crises and Monetary Policy in an Economy with Credit Constraints," *European Economic Review* 45(7), 1121–1150.
- Aguilar, Mark, and Gita Gopinath. 2007. "Emerging Market Business Cycles: The Cycle is the Trend," *Journal of Political Economy* 115, 69-102.
- Aiyagari, S. Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics* 109 (3), 659-684.
- Akinci, Ozge. 2011. "A Note on the Estimation of the Atemporal Elasticity of Substitution Between Tradable and Nontradable Goods," manuscript, Columbia University.
- Alfaro, Laura, and Fabio Kanczuk. 2009. "Optimal Reserve Management and Sovereign Debt," *Journal of International Economics* 77.1: 23-36.
- Almeida, Heitor, and Murillo Campello. 2007. "Financial Constraints, Asset Tangibility, and Corporate Investment." *Review of Financial Studies*, 20: 1429–1460.
- An, Sungbae, and Frank Schorfheide. 2007. "Bayesian Analysis of DSGE Models." *Econometric Reviews*, 26(2–4): 113–172.
- Andolfatto, David. 1996. "Business Cycles and Labor-Market Search." *American Economic Review*, 86(1): 112–132.
- Aravena, Claudio, and Juan Alberto Fuentes. 2013. "El Desempeno Mediocre de la Productividad Laboral en America Latina: Una Interpretacion Neoclasica," *Macroeconomia del Desarrollo*, 137, CEPAL, Naciones Unidas.
- Arellano, Cristina, Yan Bai, and Patrick Kehoe. 2012. "Financial Frictions and Fluctuations in Volatility." Federal Reserve Bank of Minneapolis. Research Department Staff Report 466.

- Arellano, Cristina, Yan Bai, and Jing Zhang. 2012. "Firm Dynamics and Financial Development." *Journal of Monetary Economics*, 59(6): 533–549.
- Backus, David, Patrick J. Kehoe, and Finn E. Kydland. 1993. "International Business Cycles: Theory and Evidence" NBER working paper 4493.
- Ball, Laurence M. 2009. "Hysteresis in Unemployment: Old and New Evidence." NBER. Working paper 14818.
- Ball, Laurence M., Daniel Leigh, and Prakash Loungani. 2013. "Okun's Law: Fit at Fifty?" NBER. Working paper 18668.
- Bai, Yan, Jose-Victor Rios-Rull, and Kjetil Storesletten. 2012. "Demand shocks as productivity shocks." Federal Reserve Board of Minneapolis.
- Barattieri, Alessandro, Susanto Basu, and Peter Gottschalk. 2010. "Some Evidence on the Importance of Sticky Wages," NBER working paper 16130.
- Basu, Susanto, John Fernald, Jonas Fisher, and Miles Kimball. 2013. "Sector-Specific Technical Change." National Bureau of Economic Research. Manuscript.
- Basu, Susanto, John Fernald, and Miles Kimball. 2006. "Are Technology Improvements Contractionary?" *American Economic Review*, 96(5): 1418–1448.
- Bellman, Richard E.. 1957. *Dynamic programming*, Princeton University Press, Princeton NJ.
- Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric R. Young. 2011. "Revisiting Overborrowing and its Policy Implications," in Cepedes, Luis Felipe, Roberto Chang, and Diego Saravia (Eds.), *Monetary Policy under Financial Turbulence*, Central Bank of Chile, 145-184.
- Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric R. Young. 2012a. "Financial Crises and Macro-Prudential Policies," *Journal of International Economics*, 89(2): 453-470.
- Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric R. Young. 2012b. "Capital Controls or Exchange Rate Policy? A Pecuniary Externality Perspective," CEP Discussion Paper 1160.
- Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric R. Young. 2012c. "Optimal Policy for Macro-Financial Stability," Federal Reserve Bank of Saint Louis, Working Paper 2012-041A.
- Benigno, Gianluca, and Luca Fornaro. 2012. "Reserve Accumulation, Growth and Financial Crises," CEP Discussion Paper 1161.
- Berger, David. 2012. "Countercyclical Restructuring and Jobless Recoveries," Yale University. Mimeograph.

- Bernal-Verdugo, Lorenzo, Davide Furceri, and Dominique Guillaume. 2012. "Crises, Labor Market Policy, and Unemployment," IMF. Working paper No. 12/65.
- Bernanke, Ben S., Mark Gertler, and Simon Gilchrist. 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework." In *Handbook of Monetary Economics*. Vol. 1C, ed. John B. Taylor and Michael Woodford, 1341–1393. Amsterdam: North-Holland.
- Bertola, Giuseppe, Francine D. Blau, and Lawrence M. Kahn. 2002. "Labour Market Institutions and Demographic Employment Patterns," NBER. Working paper 9043.
- Betancourt, Roger R., and Christopher K. Clague. 2008. *Capital Utilization*. Cambridge, UK: Cambridge Books.
- Bianchi, Javier. 2011. "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, 101(7): 3400-3426.
- Bianchi, Javier, Juan Carlos Hatchondo, and Leonardo Martinez. 2013. "International reserves and rollover risk" Federal Reserve Bank of Richmond, WP 13-01R.
- Bianchi, Javier, and Enrique G. Mendoza. 2011. "Overborrowing, Financial Crises and 'Macro-Prudential' Policy?," IMF Working Papers, 1-53.
- Bigio, Saki. 2014. "Endogenous Liquidity and the Business Cycle." *American Economic Review*, forthcoming.
- Blanchard, Olivier 2006. "European Unemployment: The Evolution of Facts and Ideas." *Economic Policy*, 21(45): 5–59.
- Blanchard, Olivier, and Lawrence H. Summers. 1986. "Hysteresis and the European Unemployment Problem." In *NBER Macroeconomics Annual 1986*. Vol. 1, 15–90. MIT Press.
- Bloom, Nicholas. 2009. "The Impact of Uncertainty Shocks." *Econometrica*, 77(3): 623–685.
- Bloom, Nicholas, Stephen Bond, and John Van Reenen. 2007. "Uncertainty and Investment Dynamics." *Review of Economic Studies*, 74(2): 391–415.
- Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry. 2012. "Really Uncertain Business Cycles." National Bureau of Economic Research. No. w18245.
- Botero, Juan Carlos, Simeon Djankov, Rafael La Porta, Florencio Lopez de Silanes, and Andrei Shleifer. 2004. "The Regulation of Labor." *Quarterly Journal of Economics*, 119: 1339–1382.
- Braggion, Fabio, Lawrence J. Christiano, and Jorge Roldos. 2009. "Optimal Monetary Policy in a 'Sudden Stop'," *Journal of Monetary Economics*, 56(4): 582-595.

- Brunnermeier, Markus K., Thomas M. Eisenbach, and Yuliy Sannikov. 2012. "Macroeconomics with Financial Frictions: A Survey." In *Advances in Economics and Econometrics: Tenth World Congress of the Econometric Society*. Vol. 2, ed. D. Acemoglu, M. Arellano and E. Dekel, 3–94. Cambridge, UK: Cambridge University Press.
- Bureau of Labor Statistics. 2014. "Labor Force Statistics from the Current Population Survey." <http://www.bls.gov/cps/lfcharacteristics.htm>.
- Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo. 1993. "Labor Hoarding and the Business Cycle." *Journal of Political Economy*, 101(2): 245–273.
- Caballero, Ricardo J. 1999. "Aggregate Investment." In *Handbook of Macroeconomics*. Vol. 1, ed. John B. Taylor and Michael Woodford, 813–862. Amsterdam: Elsevier.
- Calvo, Guillermo A. 1975. "Efficient and Optimal Utilization of Capital Services." *American Economic Review*, 65(1): 181–186.
- Calvo, Guillermo A.. 1998. "Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops," *Journal of Applied Economics*, 1(1): 35–54.
- Calvo, Guillermo A.. 1999. "Fixed vs. Flexible Exchange Rates: Preliminaries of a Turn-of-Millennium Rematch," manuscript, University of Maryland, 1999.
- Calvo, Guillermo A.. 2001. "Capital Markets and the Exchange Rate, with Special Reference to the Dollarization Debate in Latin America," *Journal of Money, Credit and Banking*, 33(2): 312–334.
- Calvo, Guillermo A. 2011. "The Liquidity Approach to Bubbles, Crises, Jobless Recoveries, and Involuntary Unemployment,." Columbia University. Mimeograph.
- Calvo, Guillermo A.. 2013. "Monetary Policy Challenges in Emerging Markets Integrated to the World Capital Market: Sudden Stop, Liability Dollarization, and Lender of Last Resort," in Devlin, Robert, Jose Luis Machinea and Oscar Echevarria, (Eds.), *Essays in Honor of Enrique V. Iglesias*, forthcoming.
- Calvo, Guillermo A., Fabrizio Coricelli, and Pablo Ottonello. 2012. "Labor Market, Financial Crises and Inflation: Jobless and Wageless Recoveries." National Bureau of Economic Research. paper w18480.
- Calvo, Guillermo A., Fabrizio Coricelli, and Pablo Ottonello. 2014. "Jobless Recoveries during Financial Crises: Is Inflation the Way Out?" In *Macroeconomic and Financial Stability: Challenges for Monetary Policy*. , ed. S. Bauducco, L. Christiano and C. Raddatz, forthcoming. Central Bank of Chile.
- Calvo, Guillermo A., Alejandro Izquierdo, and Ernesto Talvi. 2006. "Sudden Stops and Phoenix Miracles in Emerging Markets." *American Economic Review*, 96(2): 405–410.

- Calvo, Guillermo A., Alejandro Izquierdo, and Luis F. Mejia. 2008. "Systemic Sudden Stop: The Relevance of Balance-Sheet Effects and Financial Integration," NBER working paper 14026.
- Calvo, Guillermo A., and Carmen M. Reinhart. 2002. "Fear of Floating," *Quarterly Journal of Economics*, 117(2): 379-408.
- Campos, Nauro F., and Jeffrey B. Nugent. 2012. "The Dynamics of the Regulation of Labor in Developing and Developed Countries since 1960." University of Southern California. Mimeograph.
- Caplin, Andrew, and John Leahy. 2011. "Trading frictions and house price dynamics." *Journal of Money, Credit and Banking*, 43(s2): 283-303.
- Carlstrom, Charles T., and Timothy S. Fuerst. 1997. "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis." *American Economic Review*, 87(5): 893-910.
- Cerra, Valerie, and Sweta Chaman Saxena. 2008. "Growth Dynamics: The Myth of Economic Recovery." *American Economic Review*, 98(1): 439-457.
- Cesa-Bianchi, Ambrogio, Luis Felipe Céspedes, and Alessandro Rebucci. 2014. "Capital Flows, House Prices, and the Macroeconomy: Evidence from Advanced and Emerging Market Economies." Federal Reserve Bank of Dallas. Mimeograph.
- Céspedes, Luis Felipe, Roberto Chang and Andres Velasco. 2004. "Balance Sheets and Exchange Rate Policy," *American Economic Review*, 94(4), 1183-93.
- Chari, Varadarajan V., Patrick J. Kehoe, and Ellen R. McGrattan. 2007. "Business Cycle Accounting." *Econometrica*, 75(3): 781-836.
- Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. 2014. "Risk Shocks." *American Economic Review*, 104(1): 27-65.
- Clarida, Richard, Jordi Galí, and Mark Gertler. 2001. "Optimal Monetary Policy in Open versus Closed Economies: An Integrated Approach," *American Economic Review* Vol. 91, No. 2, Papers and Proceedings of the Hundred Thirteenth Annual Meeting of the American Economic Association, pp. 248-252.
- Cook, David. 2004. "Monetary Policy in Emerging Markets: Can Liability Dollarization Explain Contractionary Devaluations?," *Journal of Monetary Economics*, 51(6), 1155-1181.
- Cooley, Thomas F., Gary D. Hansen, and Edward C. Prescott. 1995. "Equilibrium Business Cycles with Idle Resources and Variable Capacity Utilization." *Economic Theory*, 6(1): 35-49.
- Cooper, Russell W., and John C. Haltiwanger. 2006. "On the Nature of Capital Adjustment Costs." *The Review of Economic Studies*, 73(3): 611-633.

- Corsetti, Giancarlo, and Paolo Pesenti. 2005. "International Dimensions of Optimal Monetary Policy," *Journal of Monetary economics* 52(2): 281-305.
- Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc. 2010. "Optimal Monetary Policy in Open Economies" in Friedman, Benjamin M., and Michael Woodford (Eds), *Handbook of monetary economics*. Vol. III.
- Cruces, Juan J., and Christoph Trebesch. 2013. "Sovereign Defaults: The Price of Haircuts," *American Economic Journal: Macroeconomics*, forthcoming.
- Cui, Wei. 2013. "Delayed Capital Reallocation." University College London. Manuscript.
- Curdia, Vasco. 2009. "Optimal Monetary Policy Under Sudden Stops," FRB of New York Staff Report 323.
- Daly, Mary, Bart Hobijn, and Brian Lucking. 2012. "Why Has Wage Growth Stayed Strong?," *FRBSF Economic Letter* 1-5.
- De Paoli, Bianca. 2009. "Monetary Policy and Welfare in a Small Open Economy," *Journal of International Economics* 77(1), 11-22.
- Del Negro, Marco, Marc Giannoni, and Christina Patterson. 2012. "The Forward Guidance Puzzle." Federal Reserve Bank of New York. Staff Report No. 574.
- Devereux, Michael B., and Charles Engel. 2003. "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-rate Flexibility," *The Review of Economic Studies* 70(4), 765-783.
- Devereux, Michael B., Philip R. Lane, and Juanyi Xu. 2006. "Exchange Rates and Monetary Policy in Emerging Market Economies," *The Economic Journal*, 116(511), 478-506.
- Diaz-Alejandro, Carlos F.. 1965. *Exchange Rate Devaluations in a Semi-Industrialized Country. The Experience of Argentina 1955-1961*, MIT Press.
- Dmitriev, Mikhail, and Jonathan Hoddenbagh. 2013. "The Robustness of the Financial Accelerator in Costly State Verification Models." Boston College. mimeo.
- Dornbusch, Rudiger, and Stanley Fischer. 1993. "Moderate Inflation." *The World Bank Economic Review*, 7(1): 1-44.
- Dromel, Nicolas, Elie Kolakez, and Etienne Lehmann. 2009. "Credit Constraints and Unemployment Persistence." IZA. Discussion paper no. 4501.
- Durdu, Bora, Enrique Mendoza, and Marco Terrones. 2009. "Precautionary Demand for Foreign Assets in Sudden Stop Economies: An Assessment of the New Mercantilism," *Journal of Development Economics*, 89: 194-209.

- Eggertsson, Gauti B., and Neil R. Mehrotra. 2014. "A Model of Secular Stagnation." National Bureau of Economic Research. No. 20574.
- Eichengreen, Barry, Ricardo Hausmann, and Hugo Panizza. 2005. "The Pain of Original Sin," in Eichengreen, Barry, and Ricardo Hausmann (Eds.), *Other People's Money: Debt Denomination and Financial Instability in Emerging Market Economies*, The University of Chicago Press, 13-47.
- Eichengreen, Barry, Poonam Gupta, and Ashoka Mody. 2006. "Sudden Stops and IMF-Supported Programs," NBER working paper 12235.
- Eisfeldt, Andrea L., and Adriano A. Rampini. 2006. "Capital reallocation and liquidity." *Journal of Monetary Economics*, 53(3): 369–399.
- Federal Reserve Board. 2014. "Industrial Production and Capacity Utilization – G.17: Capacity Utilization Explanatory Notes." <http://www.federalreserve.gov/releases/g17/CapNotes.htm>.
- Feenstra, Robert C., Robert Inklaar, and Marcel P. Timmer. 2013. "The Next Generation of the Penn World Table." Available for download at www.ggdc.net/pwt.
- Feldstein, Martin S.. 2011. "The Euro and European Economic Conditions," NBER working paper 17617.
- Fernald, John 2012. "A quarterly, utilization-adjusted series on total factor productivity," Federal Reserve Bank of San Francisco. Mimeograph.
- Fischer, Stanley. 1998. "The IMF and the Asian Crisis," Forum Funds Lecture at UCLA, Los Angeles, March 20.
- Fisher, Irving. 1933. "The Debt-Deflation Theory of Great Depressions," *Econometrica* 1(4): 337-357.
- Fornaro, Luca. 2013. "Financial Crises and Exchange Rate Policy," manuscript, CREI.
- Furceri, Davide, and Annabelle Mourougane. 2009. "The Effects of Financial Crises on Potential Output: New Empirical Evidence from OECD Countries,," OECD Economic Department. Working paper No. 669.
- Galí, Jordi 2011. *Unemployment Fluctuations and Stabilization Policies: A New Keynesian Perspective*. MIT Press.
- Galí, Jordi, and Tommaso Monacelli. 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy, " *The Review of Economic Studies* 72(3), 707-734.
- Galí, Jordi, Frank Smets, and Rafael Wouters. 2012. "Slow Recoveries: A Structural Interpretation." *Journal of Money, Credit and Banking*, 44: 9–30.

- Geanakoplos, John. 2010. "The Leverage Cycle." *NBER Macroeconomics Annual* 2009, 24: 1–65.
- Gertler, Mark, Simon Gilchrist, and Fabio M. Natalucci. 2007. "External Constraints on Monetary Policy and the Financial Accelerator," *Journal of Money, Credit and Banking*, Vol 39, No. 2-3, 295-33.
- Gertler, Mark, and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics*, 58(1): 17–34.
- Gertler, Mark, and Nobuhiro Kiyotaki. 2013. "Banking, Liquidity and Bank Runs in an Infinite-Horizon Economy." National Bureau of Economic Research. No. w19129.
- Gilchrist, Simon, Jae W. Sim, and Egon Zakrajšek. 2014. "Uncertainty, Financial Frictions, and Investment Dynamics." National Bureau of Economic Research. No. w20038.
- Gilchrist, Simon, and John C. Williams. 2000. "Putty–Clay and Investment: A Business Cycle Analysis." *Journal of Political Economy*, 108(5): 928–960.
- Gonzalez Rozada, Martin, Pablo A. Neumeyer, Alejandra Clemente, Diego L. Sasson, and Nicolas Trachter. 2004. "The Elasticity of Substitution in Demand for Non-Tradable Goods in Latin America: The Case of Argentina," Inter-American Development Bank, Research Network working paper R-485.
- Gorodnichenko, Yuriy, and Matthew D. Shapiro. 2011. "Using the Survey of Plant Capacity to Measure Capital Utilization." U.S. Census Bureau Center for Economic Studies. Discussion Paper.
- Gottschalk, Peter. 2005. "Downward Nominal-Wage Flexibility: Real or Measurement Error?," *The Review of Economic and Statistics* 87, 556-568.
- Gourinchas, Pierre-Olivier, Rodrigo Valdes, and Oscar Landerretche. 2001. "Lending Booms: Latin America and the World." *Journal of LACEA Economia*, 1(2): Article 3.
- Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman. 1988. "Investment, Capacity Utilization, and the Real Business Cycle." *American Economic Review*, 78(3): 402–417.
- Guerrieri, Veronica, and Robert Shimer. 2014. "Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality." *American Economic Review*, 104(7): 1875–1908.
- Hall, Robert E. 2014. "Quantifying the Lasting Harm to the U.S. Economy from the Financial Crisis." In *NBER Macroeconomics Annual*. Vol. 29, ed. Jonathan Parker and Michael Woodford, forthcoming. Cambridge, MA: National Bureau of Economic Research.

- Hansen, Bruce E.. 2000. "Sample Splitting and Threshold Estimation." *Econometrica*, 68(3): 575–603.
- Hart, Olivier, and John Moore. 1994. "A Theory of Debt Based on the Inalienability of Human Capital." *Quarterly Journal of Economics*, 109(4): 841–879.
- Hodrick, Robert J., and Edward C. Prescott. 1997. "Postwar U.S. Business Cycles: An Empirical Investigation." *Journal of Money, Credit and Banking*, 29: 1–16.
- Ilzetzi, Ethan, Carmen M. Reinhart, and Kenneth S. Rogoff. 2010. "Exchange Rate Arrangements Entering the 21st Century: Which Anchor Will Hold?," manuscript, LSE.
- Irwin, Douglas A.. 2011. *Trade Policy Disaster. Lessons from the 1930s*, MIT press.
- Jaimovich, Nir, and Henry E. Siu. 2012. "The Trend Is the Cycle: Job Polarization and Jobless Recoveries,," NBER. Working paper 18334.
- Jappelli, Tullio. 1990. "Who is Credit Constrained in the US Economy?, " *The Quarterly Journal of Economics*, 219-234.
- Jeanne, Olivier, and Romain Ranciere. 2011. "The Optimal Level of International Reserves For Emerging Market Countries: A New Formula and Some Applications." *The Economic Journal* 121.555: 905-930.
- Jermann, Urban, and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks." *American Economic Review*, 102: 238–271.
- Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti. 2010. "Investment Shocks and Business Cycles." *Journal of Monetary Economics*, 57(2): 132–145.
- Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti. 2011. "Investment Shocks and the Relative Price of Investment." *Review of Economic Dynamics*, 14(1): 102–121.
- Kaas, Leo, and Philipp Kircher. 2013. "Efficient firm dynamics in a frictional labor market." Society for Economic Dynamics. 2013 Meeting Papers, no. 160.
- Keynes, John Maynard. 1925. "The Economic Consequences of Mr. Churchill," in Moggridge, Donald (Ed.), *Collected Writings of John Maynard Keynes*, Vol. IX, 1972, New York: St. Martin's Press, 207-230.
- Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit cycles." *The Journal of Political Economy*, 105(2): 211–248.
- Korinek, Anton. 2011. "Excessive Dollar Borrowing in Emerging Markets: Balance Sheet Effects and Macroeconomic Externalities," manuscript, University of Maryland.

- Korinek, Anton, and Enrique Mendoza. 2013. "From Sudden Stops to Fisherian Deflation," NBER working paper 19362.
- Knotek, Edward S., and Stephen J. Terry. 2009. "How Will Unemployment Fare Following the Recession?" *Economic Review*, 2009(3): 5–33.
- Krainer, John. 2001. "A theory of liquidity in residential real estate markets." *Journal of Urban Economics*, 49(1): 32–53.
- Krugman, Paul. 2010. "The Euro Trap," *New York Times*, 29.
- Kurmann, André, and Nicolas Petrosky-Nadeau. 2007. "Search Frictions in Physical Capital Markets as a Propagation Mechanism." Centre Interuniversitaire sure le Risque, les Politiques Économiques and l'Emploi. Working Paper 07-12.
- Kydland, Finn E., and Edward C. Prescott. 1977. "Rules rather than discretion: The inconsistency of optimal plans," *The Journal of Political Economy*: 473-491.
- Kydland, Finn E., and Carlos E. J. Zarazaga. 2012. "Fiscal Sentiment and the Weak Recovery from the Great Recession: A Quantitative Exploration." Society for Economic Dynamics. Meeting Papers no. 1139.
- Lagos, Ricardo, Guillaume Rocheteau, and Randall Wright. 2014. "The art of monetary theory: A new monetarist perspective." mimeo.
- Lane, Phillip R, and Gian Maria Milesi-Ferreti. 2007. "The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970-2004," *Journal of International Economics*, 73: 223-250.
- Lorenzo Fernando, Diego Aboal, and Rosa Osimani. 2005. "The Elasticity of Substitution in Demand for Non-Tradable Goods in Latin America: The Case of Uruguay," Inter-American Development Bank, Research Network working paper R-480.
- Marcet, Albert, and Ramon Marimon. 2011. "Recursive Contracts," Barcelona GSE working paper 552.
- Mendoza, Enrique G.. 1991. "Real Business Cycles in a Small Open Economy," *American Economic Review*, 81(4): 797-818.
- Mendoza, Enrique G.. 1995. "The Terms of Trade, the Real Exchange Rate, and Economic Fluctuations," *International Economic Review*, 36(1): 101–37.
- Mendoza, Enrique G.. 2002. "Credit, Prices, and Crashes: Business Cycles with a Sudden Stop," in Edwards, Sebastian, and Jeffrey A. Frankel (Eds.), *Preventing Currency Crises in Emerging Markets*, Chicago: University of Chicago Press.
- Mendoza, Enrique G.. 2005. "Real Exchange Rate Volatility and the Price of Non-tradables in Sudden-Stop Prone Economies," *Economia*, Fall: 103-148.

- Mendoza, Enrique G. 2010. "Sudden stops, financial crises, and leverage." *American Economic Review*, 100(5): 1941–1966.
- Mendoza, Enrique G., and Marco E. Terrones. 2012. "An Anatomy of Credit Booms and Their Demise,," NBER. Working paper 18379.
- Mendoza, Enrique G., and Vivian Z. Yue. 2012. "A General Equilibrium Model of Sovereign Default and Business Cycles," *The Quarterly Journal of Economics* 127.2: 889-946.
- Menzio, Guido, and Shouyong Shi. 2010. "Block Recursive Equilibria for Stochastic Models of Search on the Job." *Journal of Economic Theory*, 145: 1453–1494.
- Menzio, Guido, and Shouyong Shi. 2011. "Efficient Search on the Job and the Business Cycle." *Journal of Political Economy*, 119(3): 468–510.
- Menzio, Guido, Shouyong Shi, and Hongfei Sun. 2013. "A Monetary Theory with Non-degenerate Distributions." *Journal of Economic Theory*, 148(6): 2266–2312.
- Merz, Monika. 1995. "Search in the Labor Market and the Real Business Cycle." *Journal of Monetary Economics*, 36(2): 269–300.
- Michaillat, Pascal, and Emmanuel Saez. 2013. "Aggregate Demand, Idle Time, and Unemployment." National Bureau of Economic Research. Working Paper No. 18826.
- Midrigan, Virgiliu, and Daniel Yi Xu. 2014. "Finance and Misallocation: Evidence from Plant-Level Data." *American Economic Review*, 104(2): 422–458.
- Moen, Espen R.. 1997. "Competitive Search Equilibrium." *Journal of Political Economy*, 105: 385–411.
- Monacelli, Tommaso. 2008. "Optimal Monetary Policy with Collateralized Household Debt and Borrowing Constraints," in John Y. Campbell (Ed.) *Asset Prices and Monetary Policy*, University of Chicago Press, 103-146.
- Neumeyer, Pablo A. and Fabrizio Perri. 2005. "Business Cycles in Emerging Markets: The Role of Interest Rates," *Journal of Monetary Economics*, 52: 345-380.
- Ortiz, Alberto, Pablo Ottonello, Federico Sturzenegger, and Ernesto Talvi. 2009. "Monetary and Fiscal Policies in a Sudden Stop: Is Tighter Brighter?," in Cavallo, E. and A. Izquierdo, *Dealing with International Credit Crunch: Policy Responses to Sudden Stops*, IADB, 23-73.
- Ostry, Jonathan D., and Carmen M. Reinhart. 1992. "Private Saving and Terms of Trade Shocks: Evidence from Developing Countries," IMF Staff Papers 39(3): 495–517.
- Petrosky-Nadeau, Nicolas, and Etienne Wasmer. 2011. "Macroeconomic Dynamics in a Model of Goods, Labor and Credit Market Frictions." Institute for the Study of Labor. Discussion Paper 5763.

- Piazzesi, Monika, Martin Schneider, and Johannes Stroebe. 2013. "Segmented Housing Search." Stanford University. Working paper.
- Queralto, Albert. 2013. "A Model of Slow Recoveries from Financial Crises." Federal Reserve Bank. International Finance Discussion Paper 1097.
- Ramey, Valerie A., and Matthew D. Shapiro. 1998. "Capital Churning." University of California, San Diego, Department of Economics. Mimeo.
- Ravn, Morten, and Harald Uhlig. 2002. "On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations." *Review of Economics and Statistics*, 84(2): 371–376.
- Reinhart, Carmen M., and Kenneth S. Rogoff. 2009a. *This Time Is Different: Eight Centuries of Financial Folly*. Princeton, NJ: Princeton University Press.
- Reinhart, Carmen M., and Kenneth S. Rogoff. 2009b. "The Aftermath of Financial Crises." *American Economic Review, American Economic Association*, 99(2): 466–472.
- Reinhart, Carmen M., and Kenneth S. Rogoff. 2014. "Recovery from Financial Crises: Evidence from 100 Episodes." *American Economic Review*, 104(5): 50–55.
- Reinhart, Carmen M., and Vincent Reinhart. 2010. "After the Fall,." NBER. Working paper 16334.
- Reinhart, Carmen M., and Carlos A. Vegh. 1995. "Nominal Interest Rates, Consumption Booms, and Lack of Credibility: A Quantitative Examination," *Journal of Development Economics*, 46, 357–378.
- Rognlie, Matthew, Andrei Shleifer, and Alp Simsek. 2014. "Investment Overhang and the Great Recession." Massachusetts Institute of Technology, Economics. No. w20569.
- Sargent, Thomas, Noa Williams, and Tao Zha. 2009. "The Conquest of South American Inflation," *Journal of Political Economy* 117(2): 211.
- Sbordone, Argia M.. 1996. "Cyclical Productivity in a Model of Labor Hoarding." *Journal of Monetary Economics*, 38(2): 331–361.
- Scarpetta, Stefano 1996. "Assessing the Role of Labour Market Policies and Institutional Settings on Unemployment: A Cross Country Study." *OECD Economic Studies*, 1996(26): 43–97.
- Schaal, Edouard. 2012. "Uncertainty, Productivity and Unemployment in the Great Recession." Federal Reserve Bank of Minneapolis. mimeo.
- item[]Schmitt-Grohé, Stephanie, and Martin Uribe. 2001. "Stabilization policy and the costs of dollarization," *Journal of Money, Credit and Banking*: 482–509.
- Schmitt-Grohé, Stephanie, and Martin Uribe. 2011. "Pegs and Pain," NBER working paper 16847.

- Schmitt-Grohé, Stephanie, and Martin Uribe. 2012a. "What's News in Business Cycles." *Econometrica*, 80(6): 2733–2764.
- Schmitt-Grohé, Stephanie, and Martin Uribe. 2012b. "The Making of a Great Contraction with a Liquidity Trap and a Jobless Recovery." National Bureau of Economic Research. No. w18544.
- Schmitt-Grohé, Stephanie, and Martin Uribe. 2013. "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," manuscript, Columbia University. Mimeograph.
- Schreft, Stacey L., Aarti Singh, and Ashley Hodgson. 2005. "Jobless Recoveries and the Wait-and-See Hypothesis." *Economic Review*, 90(4): 81.
- Schularick, Moritz, and Alan Taylor. 2012. "Credit Booms Gone Bust: Monetary Policy, Leverage Cycles and Financial Crises, 1870–2008." *American Economic Review*, 102(2): 1029–1061.
- Schwartzman, Felipe. 2012. "When do Credit Frictions Matter for Business Cycles?" *Economic Quarterly*, 98(3): 209–230.
- Shi, Shouyong. 2009. "Directed Search for Equilibrium Wage Tenure Contracts." *Econometrica*, 77(2): 561–584.
- Shimer, Robert. 1996. "Contracts in a Frictional Labor Market." Massachusetts Institute of Technology. mimeo.
- Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review*, 95(1): 25–49.
- Shimer, Robert. 2012. "Wage Rigidities and Jobless Recoveries." *Journal of Monetary Economics*, 59: S65–S77.
- Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in U.S. Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97(3): 586–606.
- Stiglitz, Joseph. 2002. *Globalization and Its Discontents*. New York: WW Norton.
- Stockman, Alan, and Linda Tesar. 1995. "Tastes and Technology in a Two Country Model of the Business Cycle: Explaining International Comovements," *American Economic Review* 85(1), 168–85.
- Svensson, Lars E. O.. 2010. "Optimization under Commitment and Discretion, the Recursive Saddlepoint Method, and Targeting Rules and Instrument Rules: Lecture Notes," manuscript, Stockholm University.
- Tauchen, George. 1986. "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economic Letters*, 20(2): 177–181.

- Terry, Stephen J. and Edward S. Knoteck II. 2011. "Markov-Chain Approximations of Vector Autoregressions: Application of General Multivariate-Normal Integration Techniques," *Economic Letters* 110(1): 4-6.
- Tirole, Jean 2005. *The Theory of Corporate Finance*. Princeton University Press.
- Townsend, Robert M. 1979. "Optimal Contracts and Competitive Markets with Costly State Verification," *Journal of Economic Theory*, 21(2): 265–293.
- Uribe, Martin. 1997. "Exchange-Rate-Based Inflation Stabilization: The Initial Real Effects of Credible Plans," *Journal of Monetary Economics*, 39: 197-221.
- Uribe, Martin, and Stephanie Schmitt-Grohé. 2014. *Open Economy Macroeconomics*. Columbia University.
- Uribe, Martin, and Vivian Yue. 2006. "Country Spreads and Emerging Countries: Who Drives Whom?," *Journal of International Economics* 69(1), 6-36.
- U.S. Census Bureau. 2014. "Quarterly Survey of Plant Capacity Utilization (QPC)." <http://www.census.gov/manufacturing/capacity/>.
- Wheaton, William C. 1990. "Vacancy, search, and prices in a housing market matching model." *Journal of Political Economy*, 98(6): 1270–1292.
- Winston, Gordon C. 1974. "The Theory of Capital Utilization and Idleness." *Journal of Economic Literature*, 12(4): 1301–1320.

Appendix A

Appendix for Chapter 1

A.1 Data Appendix

A.1.1 Financial-Crises, Investment, and Capital Stock

To study investment recovery during financial crises, I construct a sample of post-WWII recession episodes in advanced economies. The sample includes annual data from 1950 to 2013 for 22 countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Taiwan, United Kingdom, United States. Only recessions prior to 2007 were considered.

A recession event is identified by a period of contraction in annual real GDP per capita (a similar empirical strategy is followed, for example, in [Calvo, Izquierdo and Talvi, 2006](#)). Given a contraction in GDP per capita, the output peak is defined as the period prior to the beginning of a recession episode; the recovery point is defined as the period in which output per capita recovers its precrisis level; the output trough is defined as the period with the lowest level of GDP per capita between output peak and recovery point.¹

¹More formally, for each country i the algorithm to identify recession episodes can be described as follows. Let y_{it} denote GDP per capita of country i in period t .

- (i) Set $t_0 = 1950$.
- (ii) Let $\Gamma_p = \{\tau \in [t_0, 2007] : y_{i,\tau} < y_{i,\tau-1}\}$. If $\Gamma_p = \emptyset$ country i has no more recession episodes. If $\Gamma_p \neq \emptyset$ set $p = \min\{\Gamma_p\} - 1$. Let $\Gamma_r = \{\tau \in [p, 2007] : y_{i,\tau} > y_{i,p}\}$. If $\Gamma_r = \emptyset$ country i has no more recession episodes. If $\Gamma_r \neq \emptyset$ set $r = \min\{\Gamma_r\}$. Denote with p the recession peak and with r the recession trough.

TABLE A.1
SAMPLE OF RECESSION EPISODES

Financial crises		Other episodes							
Country	Peak	Country	Peak	Country	Peak	Country	Peak	Country	Peak
Australia	1989	Australia	1950	Denmark	1973	Ireland	1951	Spain	1974
Canada	1981	Australia	1955	Denmark	1979	Ireland	1955	Spain	1992
Denmark	1987	Australia	1960	Finland	1952	Ireland	1965	Sweden	1976
Denmark	1992	Australia	1973	Finland	1956	Ireland	1974	Sweden	1980
Finland	1990	Australia	1976	Finland	1976	Ireland	1982	Switzerland	1951
France	1992	Australia	1981	France	1974	Italy	1974	Switzerland	1957
Greece	1989	Austria	1951	Germany	1966	Italy	2001	Switzerland	1974
Greece	1991	Austria	1974	Germany	1974	Japan	1973	Switzerland	1981
Iceland	1982	Austria	1977	Germany	1981	Netherlands	1957	Switzerland	1990
Iceland	1987	Austria	1980	Germany	1992	Netherlands	1974	Switzerland	2001
Italy	1992	Austria	1983	Germany	2001	Netherlands	1980	Taiwan	2000
Japan	1992	Austria	1992	Greece	1961	Netherlands	2001	United Kingdom	1957
Japan	1997	Belgium	1957	Greece	1973	Norway	1981	United Kingdom	1968
Norway	1987	Belgium	1974	Greece	1979	Portugal	1957	United Kingdom	1979
Spain	1978	Belgium	1980	Iceland	1950	Portugal	1973	United States	1953
Spain	1980	Belgium	1992	Iceland	1956	Portugal	1982	United States	1957
Sweden	1990	Canada	1953	Iceland	1960	Portugal	1992	United States	1969
United Kingdom	1973	Canada	1956	Iceland	1966	Portugal	2002	United States	1973
United Kingdom	1990	Canada	1989	Iceland	1974	Spain	1952	United States	1979
United States	1990	Denmark	1954	Iceland	2001	Spain	1958	United States	1981

Recession episodes are then classified into *financial crises* and *regular recession episodes*. Following [Calvo, Coricelli and Ottonello \(2012\)](#) a financial crisis is defined as a recession episode in which a banking-crisis event (as defined in [Reinhart and Rogoff, 2009a](#)) took place between the output peak and the recovery point. Regular recession episodes are recession episodes not classified as financial crises. With this methodology, a sample of 100 recession episodes is obtained, with 20 financial crises and 80 regular recession episodes (see Table A.1). For each recession episode $t = 0$ is defined as the output trough. Variables of interests are then averaged in a window around $t = 0$ (from $t = -2$ to $t = 4$).

The source of the data used to identify recession episodes and construct the time series of average recession episodes shown in Figure 1.1 was [Feenstra, Inklaar and](#)

-
- (iii) Set $t_0 = r$ and repeat from (ii) until the country has no more recession episodes.

Timmer (2013), *Penn World Tables*, downloaded from <http://www.ggdg.net/pwt>.

In particular, the following data were used:

1. Real GDP: Real GDP at constant 2005 national prices.
2. Real Capital Stock: Capital stock at constant 2005 national prices.
3. Population.
4. Real Per Capita GDP: Constructed as $(4) = (1) / (3)$.
5. Real Per Capita Capital Stock: Constructed as $(5) = (2) / (3)$.
6. Share of gross capital formation at current PPPs

The time series used in Figure 1.1 were (4), (5) (for each country, expressed in percent deviation from a log-quadratic trend), and (6) (for each country, expressed in percent deviation from its mean 1950–2013).

For the U.S. Great Recession the following time series were used for Figure 1.1:

1. Nominal GDP: Gross domestic product, billions of dollars, seasonally adjusted at annual rates. Source: U.S. Bureau of Economic Analysis (BEA, <http://www.bea.gov>), National Income and Product Accounts Tables (Table 1.1.5).
2. Nominal Investment: Gross private domestic fixed investment. Source: BEA, National Income and Product Accounts Tables (Tables 1.1.5).
3. Real GDP: Gross domestic product, billions of chained (2009) dollars, seasonally adjusted at annual rates. Source: BEA, National Income and Product Accounts Tables (Table 1.1.6).
4. Real Capital Stock: Private fixed assets, chain-type quantity indexes. Source: BEA, Fixed Assets Accounts Tables (Table 2.2).

5. Civilian Noninstitutional Population, 16 years and over. Source: Bureau of Labor Statistics, Labor Force Statistics from the Current Population Survey.
6. Real Per Capita GDP: Constructed as $(6) = (3) / (5)$.
7. Real Per Capita Capital Stock: Constructed as $(7) = (4) / (5)$.
8. Investment Rate: Constructed as $(8) = (2) / (1)$.

The time series used in Figure 1.1 were (6), (7) (expressed in percent deviation from a log-quadratic trend) and (8) (expressed in percent deviation from its mean 1950–2013).

A.1.2 Capital Unemployment

The data on capital unemployment for structures in the U.S. economy – used in Figure 1.2 and in the model estimation of Section 1.5 – were constructed as a weighted average of quarterly vacancy rates of office space, retail space, and industrial space. Data were obtained from CBRE (<http://www.cbre.com/EN/Pages/Home.aspx>) and REIS (<https://www.reis.com/>). Weights for office space, retail space, and industrial space were defined using data on Current-Cost Net Stock of Private Fixed Assets, Equipment, Structures, and Intellectual Property Products by Type, source U.S. Bureau of Economic Analysis (BEA, <http://www.bea.gov>), Table 2.1. The following items were included to compute the weight. For Office space: “Office”; for retail space: “Multimerchandise shopping”, “Food and beverage establishments”, “Commercial warehouses”, and “Other commercial”; for industrial space: “Manufacturing”, “Power and communication”, and “Mining exploration, shafts, and wells.” These items jointly represent 60.5% of nonresidential structures. The weights were computed as the average of the share of each item over the period 1980–2012, which is similar to the period for which the data on vacancy rates is available (1980–2013).

A.1.3 Bayesian Estimation

The following data for the U.S. economy were used to construct the quarterly time series used in the model estimation of Section 1.5:

1. Nominal GDP: Gross domestic product, billions of dollars, seasonally adjusted at annual rates. Source: U.S. Bureau of Economic Analysis (BEA, <http://www.bea.gov>) National Income and Product Accounts Tables (Table 1.1.5).
2. Nominal Consumption: Sum of personal consumption expenditures, durable goods and services, billions of dollars, seasonally adjusted at annual rates. Source: BEA, National Income and Product Accounts Tables (Table 1.1.5).
3. Nominal Investment: Sum of gross private domestic fixed nonresidential investment in structures, equipment and software. Source: BEA, National Income and Product Accounts Tables (Tables 1.1.5 and 5.3.5).
4. Real GDP: Gross domestic product, billions of chained (2009) dollars, seasonally adjusted at annual rates. Source: BEA, National Income and Product Accounts Tables (Table 1.1.6).
5. GDP Deflator: constructed as $(5) = (1) / (4)$.
6. Nonfarm Business Hours Worked. Source: Bureau of Labor Statistics (BLS, <http://www.bls.gov/>), Major Sector Productivity and Costs.
7. Civilian Noninstitutional Population, 16 years and over. Source: BLS, Labor Force Statistics from the Current Population Survey.
8. 3-Month Treasury Bill, Secondary Market Rate. Source: Federal Reserve Bank of St. Louis, Federal Reserve Economic Data (FRED, <http://www.stlouisfed.org/>).
9. Moody's Seasoned Baa Corporate Bond Yield. Source: FRED.

10. Real Per Capita GDP: constructed as $(10) = (4) / (7)$.
11. Real Per Capita Consumption: constructed as $(11) = ((2) / (5)) / (7)$.
12. Real Per Capita Investment: constructed as $(12) = ((3) / (5)) / (7)$.
13. Per Capita Hours Worked: constructed as $(13) = (6) / (7)$.
14. Credit Spreads: constructed as $(14) = (1 + (9)) / (1 + (8))$.
15. Capital Unemployment: constructed based on data on vacancy rates of nonresidential commercial real estate (office, retail, and industrial sectors). Methodology detailed in A.1.2.

The six time series used in the Bayesian estimation were (10), (11), (12), (13), (14) and (15), with (10), (11), (12) log-linearly detrended.

A.2 Additional Figures

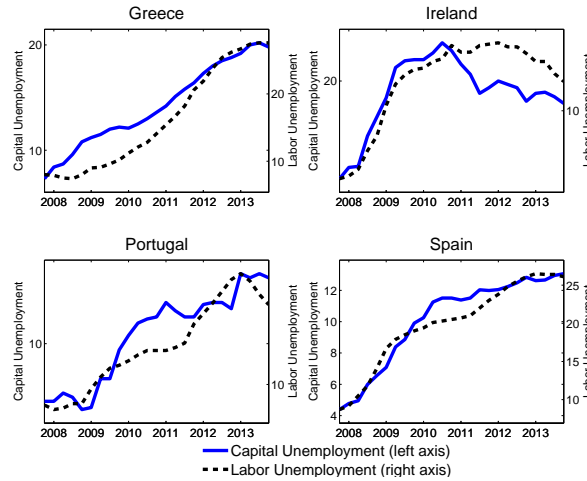


FIGURE A.1: Unemployment of Capital and Labor, Euro Economies, 2007–2013.
Note: Capital unemployment (structures) refers to the vacancy rates of office space (<http://www.jll.eu/emea/en-gb/>). Labor unemployment refers to the unemployment rate (<http://epp.eurostat.ec.europa.eu/portal/page/portal/eurostat/home/>). Data is expressed in percent.

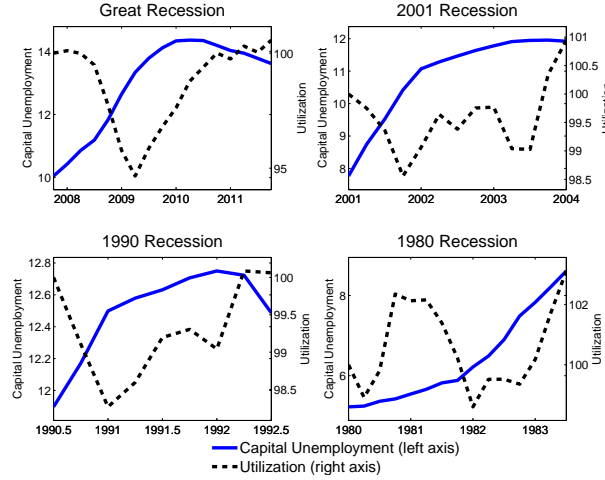


FIGURE A.2: Unemployment of Capital and Utilization, U.S. Recession Episodes.
Note: Capital unemployment (structures) constructed based on vacancy rates of office, retail and industrial units. Data source: CBRE and REIS. See Appendix A.1 for details. Data on utilization refers to estimates of factor utilization for the U.S. economy in ?, capturing labor effort and the work week of capital. Capital Unemployment expressed in percent. For each recession episode, utilization index = 100 at the output peak.

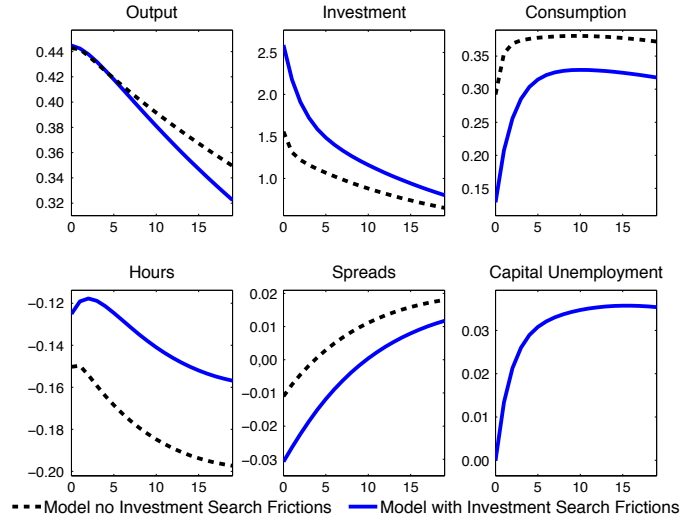


FIGURE A.3: Impulse-Responses to a Neutral-Technology Shock.

Note: Response of output, investment, consumption, hours worked, credit spreads, and capital unemployment to a one-standard-deviation neutral-technology shock (A). Label “Model with Search Frictions in Investment” and “Model No Search Frictions in Investment” refer, respectively, to the model responses presented in Section 1.4 and the benchmark model in Appendix A.5. Impulse responses expressed in percent deviations from steady state.

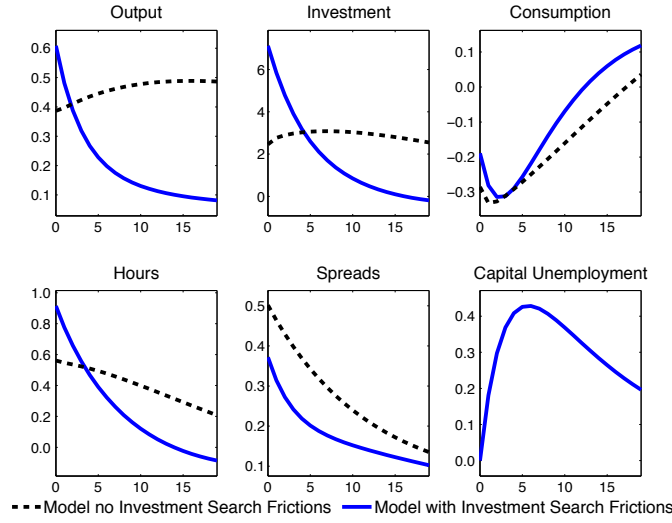


FIGURE A.4: Impulse-Responses to an Investment-Specific Technology Shock.

Note: Response of output, investment, consumption, hours worked, credit spreads, and capital unemployment to a one-standard-deviation investment-specific technology shock (A^I). Label “Model with Search Frictions in Investment” and “Model No Search Frictions in Investment” refer, respectively, to the model responses presented in Section 1.4 and the benchmark model in Appendix A.5. Impulse responses expressed in percent deviations from steady state.

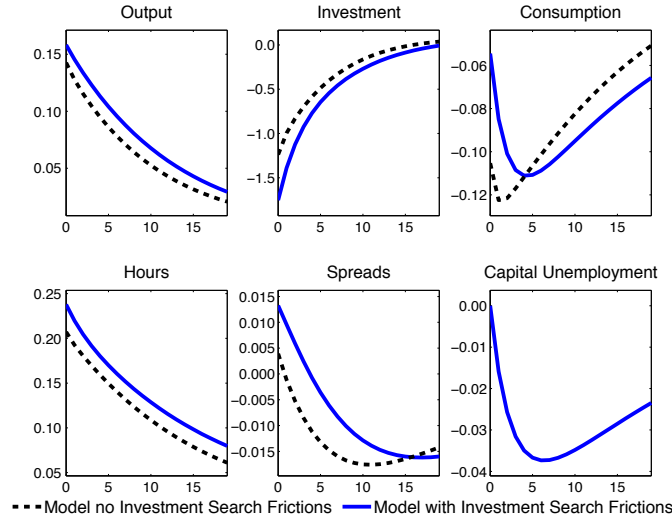


FIGURE A.5: Impulse-Responses to a Government Spending Shock.

Note: Response of output, investment, consumption, hours worked, credit spreads, and capital unemployment to a one-standard-deviation government spending shock (G). Label “Model with Search Frictions in Investment” and “Model No Search Frictions in Investment” refer, respectively, to the model responses presented in Section 1.4 and the benchmark model in Appendix A.5. Impulse responses expressed in percent deviations from steady state.

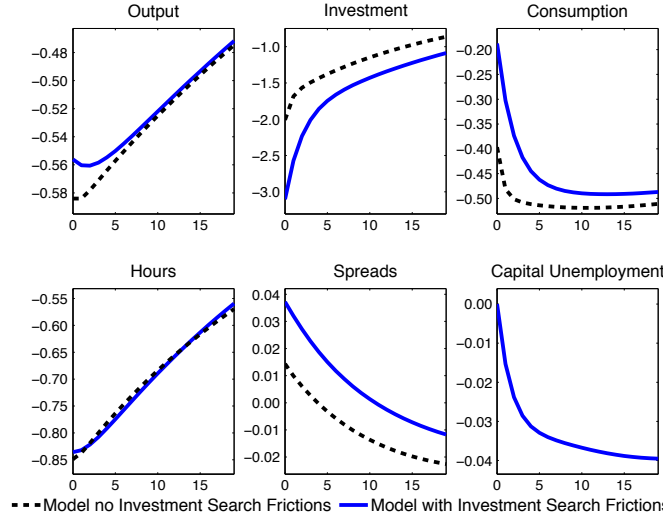


FIGURE A.6: Impulse-Responses to a Labor-Wedge Shock.

Note: Response of output, investment, consumption, hours worked, credit spreads, and capital unemployment to a one-standard-deviation labor-wedge shock (φ). Label “Model with Search Frictions in Investment” and “Model No Search Frictions in Investment” refer, respectively, to the model responses presented in Section 1.4 and the benchmark model in Appendix A.5. Impulse responses expressed in percent deviations from steady state.

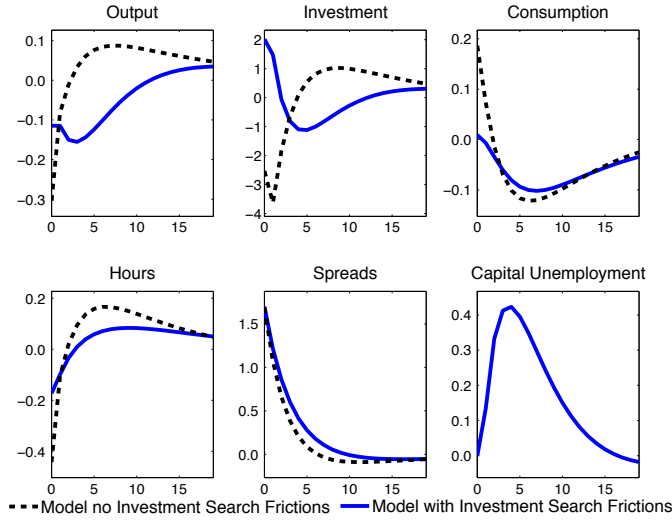


FIGURE A.7: Impulse-Responses to a Risk Shock.

Note: Response of output, investment, consumption, hours worked, credit spreads, and capital unemployment to a one-standard-deviation risk shock (σ). Label “Model with Search Frictions in Investment” and “Model No Search Frictions in Investment” refer, respectively, to the model responses presented in Section 1.4 and the benchmark model in Appendix A.5. Impulse responses expressed in percent deviations from steady state.

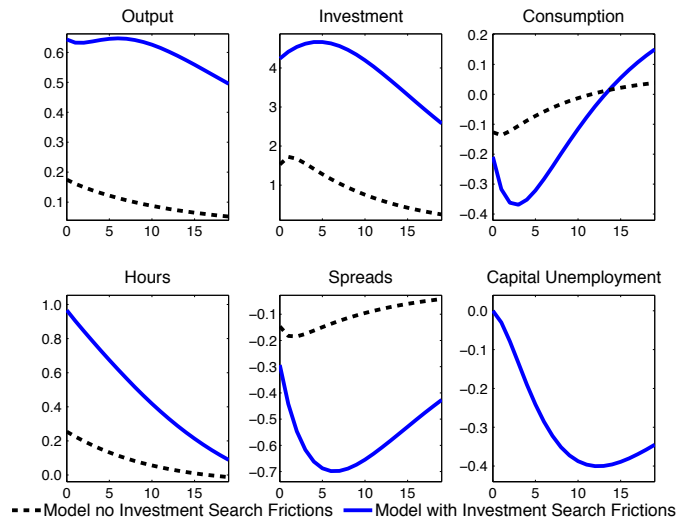


FIGURE A.8: Impulse-Responses to an Equity Shock.

Note: Response of output, investment, consumption, hours worked, credit spreads, and capital unemployment to a one-standard-deviation equity shock (ζ). Label “Model with Search Frictions in Investment” and “Model No Search Frictions in Investment” refer, respectively, to the model responses presented in Section 1.4 and the benchmark model in Appendix A.5. Impulse responses expressed in percent deviations from steady state.

A.3 Mapping from Investment Search Frictions to Wedges

To further study the economic mechanism induced by the search friction in investment, this section considers a prototype economy with time-varying wedges (in the spirit of [Chari, Kehoe and McGrattan, 2007](#)), and maps the equilibrium of the economy presented in Section 1.3 with search frictions in investment to wedges in the prototype economy.

The *prototype economy* corresponds to a neoclassical growth model, with no disutility from labor, and with time-varying exogenous productivity, taxes on capital income, and government consumption. Agents in this economy are households, firms, and the government. The household's problem in the prototype economy is given by

$$\max_{\{\hat{C}_t, \hat{I}_t, \hat{K}_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(\hat{C}_t), \quad (\text{A.1})$$

$$\text{s.t. } \hat{C}_t + \hat{I}_t + \hat{T}_t = (1 - \hat{\tau}_t^k) \hat{r}_t^k \hat{K}_t + \hat{W}_t \bar{h} + \hat{\Pi}_t^f, \quad (\text{A.2})$$

$$\hat{K}_{t+1} = (1 - \delta) \hat{K}_t + \hat{I}_t, \quad (\text{A.3})$$

where “hats” represent variables in the prototype economy; \hat{C}_t denotes consumption in period t , \hat{I}_t denotes consumption in period t , \hat{K}_t denotes the stock of capital held by households in period t , \hat{r}_t^k denotes the rental rate of capital in period t taken as given by households, $\hat{\tau}_t^k$ denotes a capital-income-tax in period t , \hat{W}_t denotes the wage rate in period t taken as given by households, \bar{h} denotes the household (inelastic) supply of hours of work to the labor market, \hat{T}_t denotes lump-sum taxes levied by the government on households in period t , and $\hat{\Pi}_t^f$ denote lump-sum transfers from the entrepreneurs to households in period t taken as given by households.

Firms rent capital and employ labor from households each period, in competitive markets, to maximize profits, given by $\hat{\Pi}_t^f \equiv \hat{A}_t F(\hat{K}_t, \hat{h}_t) - \hat{r}_t^k \hat{K}_t - \hat{W}_t \hat{h}_t$, where \hat{A}_t denotes aggregate productivity in period t .

The government budget constraint in the prototype economy is given by

$$\hat{G}_t = \hat{T}_t, \quad (\text{A.4})$$

where \hat{G}_t denotes an exogenous government consumption in period t .

Definition 7 (Equilibrium in the prototype economy with wedges). *Given initial conditions for capital, \hat{K}_0 , and a sequence of wedges $\{\hat{A}_t, \hat{G}_t$ and $\hat{\tau}_t^k\}$, an equilibrium in the prototype economy with wedges is a sequence of allocations $\{\tilde{C}_t, \tilde{I}_t, \tilde{K}_{t+1}\}$ such that three conditions are satisfied:*

$$U'(\hat{C}_t) = \beta U'(\hat{C}_{t+1}) \left[(1 - \hat{\tau}_{t+1}^k) \hat{A}_{t+1} F_1(\hat{K}_{t+1}, \bar{h}) + (1 - \delta) \right] \quad (\text{A.5})$$

$$\hat{K}_{t+1} = (1 - \delta) \hat{K}_t + \hat{I}_t \quad (\text{A.6})$$

$$\hat{C}_t + \hat{G}_t + \hat{I}_t = \hat{A}_t F(\hat{K}_t, \bar{h}). \quad (\text{A.7})$$

Equation (A.5) is the standard intertemporal optimality condition for the household's problem, with the rental rate of capital r_t^k replaced by its equilibrium value, $\hat{r}_t^k = \hat{A}_t F_1(\hat{K}_t, \bar{h})$. Equation (A.7) is the resource constraint of the prototype economy obtained by aggregating the households', firms', and government's budget constraints and using the definition of firms profits.

To establish the mapping with the economy with investment search frictions, let the efficiency wedge in the prototype economy, \hat{A}_t , be given by

$$\hat{A}_t = A_t (1 - k_t^u)^\alpha, \quad (\text{A.8})$$

where variables without “hat” denote allocations in the economy with investment search frictions (Definition 2). Let the capital-income tax in the prototype economy be implicitly defined by

$$(1 - \hat{\tau}_t^k) \hat{A}_t F_1(\hat{K}_t, \bar{h}) = p(\theta_t^u)(Q_t^{\text{sp}} - 1) - c_s \theta_t^u. \quad (\text{A.9})$$

Let government consumption in the prototype, \hat{G}_t , be given by

$$\hat{G}_t = c_s \theta_t^u (1 - \delta) k_t^u K_t. \quad (\text{A.10})$$

Then the following equivalence result can be established.

Proposition 5. *Let $\{C_t, I_t, K_{t+1}, k_{t+1}^u, \theta_t^u\}$ denote equilibrium allocations of the economy with investment search frictions (Definition 2), for given initial conditions for capital stock and capital-unemployment rate, K_0 and k_0^u , and sequences of aggregate productivity, A_t . If the efficiency wedge is given by (A.8), the capital-income-tax wedge is given by (A.9), and the government consumption wedge is given by (A.10), the allocations $\{C_t, I_t, K_{t+1}\}$ constitute an equilibrium of the prototype economy (Definition 7).*

Proof. See Appendix A.4. ■

From this proposition, it follows that the investment search frictions proposed in this section manifest themselves as three wedges in a neoclassical growth model without search frictions. First, an efficiency wedge, as shown in (A.8), is the direct result of capital unemployment, the fact that only a fraction, $1 - k_t^u$, is used for production in period t in the economy with investment search friction. Second, an investment wedge, as shown in (A.9), relates the marginal benefits of saving to the shadow value of employed capital, net of search costs. Third, a government spending wedge, as shown in (A.10), subtracts search costs from the resources available to the economy each period. It is relevant to note that the wedges of the prototype economy without search frictions, defined in (A.8)–(A.10) depend on the evolution of the endogenous state variable, k_t^u . Therefore, the allocation of other models with friction that manifest themselves as efficiency, investment or government spending wedges will generally differ from the allocation in the model economy presented in this section.

A.4 Proofs

A.4.1 Proof of Proposition 1

Using the equilibrium market-tightness function (1.17), the first-order condition for households (1.6) can be expressed as

$$-c_s \theta_t(x_t^u) q'(\theta_t(x_t^u)) = p'(\theta_t(x_t^u)) q(\theta_t(x_t^u)) (x_t^u - 1), \quad (\text{A.11})$$

From Definition 1 and equation (A.11), it follows that sequences $\{C_t, I_t, K_{t+1}^e, K_{t+1}^u, \Lambda_t, Q_t, \theta_t^u, x_t^u\}$ are a competitive equilibrium if and only if they satisfy the following conditions

$$U'(C_t) = \Lambda_t, \quad (\text{A.12})$$

$$\Lambda_t = \beta \Lambda_{t+1} (1 - \delta) \{p(\theta_{t+1}^u) x_{t+1} - (1 - p(\theta_{t+1}^u))\}, \quad (\text{A.13})$$

$$-c_s \theta_t^u q'(\theta_t^u) = p'(\theta_t^u) q(\theta_t^u) (x_t - 1), \quad (\text{A.14})$$

$$\Lambda_t Q_t = \beta \Lambda_{t+1} [A_{t+1} F_1(K_{t+1}^e, \bar{h}) + (1 - \delta) (\psi + (1 - \psi) Q_{t+1})], \quad (\text{A.15})$$

$$Q_t = x_t + \frac{c_s}{q(\theta_t^u)}, \quad (\text{A.16})$$

$$K_{t+1}^e = (1 - \psi)(1 - \delta) K_t^e + A_t^e p(\theta_t^u) (1 - \delta) K_t^u, \quad (\text{A.17})$$

$$K_{t+1}^u = (1 - \theta_t^u)(1 - \delta) K_t^u + \psi(1 - \delta) K_t^e + I_t, \quad (\text{A.18})$$

$$A_t F(K_t^e, \bar{h}) = C_t + I_t + c_s \theta_t^u (1 - \delta) K_t^u. \quad (\text{A.19})$$

To show that the competitive equilibrium is efficient, it must be shown that if sequences $\{C_t, I_t, K_{t+1}^e, K_{t+1}^u, \Lambda_t, Q_t, \theta_t^u, x_t^u\}$ satisfy (A.12)–(A.19), they also satisfy the social planner's optimality conditions (1.23)–(1.29). Replacing the definitions of capital-unemployment rate (1.20) and total capital stock in (A.17), (A.18), and (A.19), and operating, equations (1.23), (1.24), and (1.25) are obtained. Pick $\Lambda_t = \Lambda_t^{\text{sp}}$; replacing in (A.12), equation (1.26) is obtained. Pick $Q_t = Q_t^{\text{sp}}$; replacing in (A.15), equation (1.28) is obtained. Replacing (A.16) in (A.14), and operating, equation (1.28) is obtained. Finally, replacing (A.16) in (A.13), equation (1.29) is obtained.

A.4.2 Proof of Proposition 5

To establish the mapping between the economy with investment search frictions and the prototype economy with wedges, it must be shown that if sequences $\{C_t, I_t, K_{t+1}, k_{t+1}^u, \theta_t^u\}$ satisfy the social planner's optimality conditions (1.23)–(1.29), and wedges are defined by (A.8)–(A.9), then the allocations $\{C_t, I_t, K_{t+1}\}$ also satisfy (A.5)–(A.7).

Replacing the definition of the efficiency wedge, (A.8), and the definition of the government consumption wedge, (A.10), on the resource constraint of the social planner's problem, (1.23), the resource constraint of the prototype economy, (A.7), is obtained. Replacing equation (1.26) and the definition of the capital-income-tax wedge on the planner's optimality condition (A.6), equation (1.23) is obtained. Finally, the social planner's capital-accumulation constraint (1.24) coincides with the prototype economy's capital-accumulation constraint, (A.5). Therefore, equations (A.5)–(A.7) are satisfied.

A.5 Benchmark Business Cycle Economy

This section presents the benchmark business-cycle model used in Section 1.5 for comparison with the model developed in Section 1.4. The only difference between the two models is that the benchmark economy does not include investment search frictions. The notation used in this section is the same as that presented in Section 1.4.

Goods. As in Section 1.4, there are perishable consumption goods, and capital goods that depreciate at a rate $\delta > 0$. Unlike 1.4, there is no distinction between matched and unmatched capital.

Agents. As in Section 1.4, the economy is populated by a large number of identical households, entrepreneurs and financial intermediaries (see Figure 1.6).

Markets. As in Section 1.4, the economy has four competitive markets: goods, labor, physical capital and credit (see Figure 1.6). The goods and labor markets are frictionless. Unlike Section 1.4, the market for physical capital is also frictionless. In this market, households and entrepreneurs trade capital at the price Q_t . The credit market is characterized by frictions associated with asymmetric information in lending as described in Section 1.4.

Households. Household i 's problem is

$$\begin{aligned} \max_{\{C_{i,t}, I_{i,t}, B_{i,t}, h_{i,t}\}} E_0 \sum_{t=0}^{\infty} \beta^t \{U(C_{i,t} - \rho_c C_{i,t-1}) - V(h_{i,t}; \varphi_t)\}, \\ \text{s.t. } C_{i,t} + I_{i,t} + T_t + B_{i,t} = R_{t-1} B_{i,t-1} + W_t h_{i,t} + Q_t A_t^I \left[I_{i,t} - \Phi \left(\frac{I_{i,t}}{K_t} \right) K_t \right] + \Pi_t. \end{aligned}$$

The only difference with respect to the household's problem presented in Section 1.4 is that households sell their capital stock to entrepreneurs in a centralized market at the price Q_t .

Entrepreneurs. Entrepreneur j 's problem is

$$\begin{aligned} \max_{\{\tilde{h}_{j,t}, u_{j,t}, L_{j,t}, \omega_{j,t+1}\}} E_t \left\{ \left[\int_{\bar{\omega}_{j,t+1}}^{\infty} \omega dF_{\omega}(\omega; \sigma_t) - (1 - F_{\omega}(\bar{\omega}_{j,t+1}; \sigma_t)) \bar{\omega}_{j,t+1} \right] R_{j,t+1}^k L_{j,t} N_{j,t+1} \right\} \\ \text{s.t. } \frac{L_{j,t} - 1}{L_{j,t}} R_t = [1 - F_{\omega}(\bar{\omega}_{j,t+1}; \sigma_t)] \bar{\omega}_{j,t+1} R_{j,t+1}^k + (1 - \mu_m) \int_0^{\omega_{j,t+1}} \omega dF_{\omega}(\omega; \sigma_t) R_{j,t+1}^k, \\ r_{j,t}^k = \left(A_t (\tilde{h}_{j,t})^{1-\alpha} - W_t \tilde{h}_{j,t} \right) u_{j,t} - \mathcal{C}_u(u_{j,t}), \\ R_{j,t+1}^k = \frac{r_{j,t+1}^k + (1 - \delta) Q_{t+1}}{Q_t}. \end{aligned}$$

The difference with respect to the entrepreneur's problem presented in section 1.4 is that entrepreneurs only purchase capital in a centralized market at the price Q_t .

Equilibrium. In equilibrium all markets clear. Similar to Section 1.4, aggregate net worth evolves following the law of motion

$$N_{t+1} = [1 - \Gamma_{t-1}(\bar{\omega}_t)] R_t^k Q_{t-1} K_t + \zeta_t. \quad (\text{A.20})$$

The net transfer from entrepreneurs to households is given by

$$\Pi_t = [1 - \Gamma_{t-1}(\bar{\omega}_t)] R_t^k Q_{t-1} K_t - \zeta_t. \quad (\text{A.21})$$

The aggregate capital stock evolves following the law of motion

$$K_{t+1} = (1 - \delta) K_t + A_t^I \left[I_t - \Phi \left(\frac{I_t}{K_t} \right) K_t \right]. \quad (\text{A.22})$$

The economy's resource constraint is given by

$$C_t + I_t + G_t = A_t (K_t)^\alpha (h_t)^{(1-\alpha)} - \Omega_t - \mathcal{C}_u(u_t) K_t, \quad (\text{A.23})$$

where $\Omega_t \equiv \mu g_{t-1}(\bar{\omega}_t) R_t^k Q_{k,t-1} K_t$.

An equilibrium in the benchmark economy can then be defined as follows.

Definition 8 (Competitive equilibrium). *Given initial conditions for capital, K_0 and consumption C_{-1} , and a state-contingent sequence of aggregate exogenous states, S_t^x , a competitive equilibrium is a state-contingent sequence of individual allocations and shadow values,*

$\{(C_{i,t}, h_{i,t}, I_{i,t}, \iota_{i,t}^h, K_{i,t+1}^u, B_{i,t}, x_{i,t}^u)_{i \in [0,1]}, (\tilde{h}_{j,t}, u_{j,t}, L_{j,t}, \omega_{j,t+1})_{j \in [0,1]}\}, \{(\Lambda_{i,t})_{i \in [0,1]}, (Q_{j,t})_{j \in [0,1]}\}$, aggregate allocations, $\{C_t, I_t, h_t, K_{t+1}^e, K_{t+1}^u, N_t, \Pi_t\}$, prices, $\{Q_t^c, J_t^u, W_t\}$, and debt schedules $\{\mathcal{D}_t(\tilde{h}_{j,t}, u_{j,t})\}$, such that:

- (i) *Individual allocations and shadow values solve the household's and entrepreneur's problems at the equilibrium prices and debt schedules, for all i and j .*
- (ii) *Debt schedules satisfy financial intermediaries' participation constraint (1.53).*
- (iii) *All markets clear.*

Appendix B

Appendix for Chapter 2

B.1 List of Recession Episodes

Table [B.1](#) lists the recession episodes included in the empirical analysis. The identification of recession episodes and their classification into low and high inflation and financial crises and other episodes is detailed in Section [2.2](#).

TABLE B.1
SAMPLE RECESSION EPISODES

		Financial Crises				Other Episodes			
		Country	Peak	Country	Peak	Country	Peak	Country	Peak
Developed Market Economies									
		Australia	1990			Austria	1974	Italy	1980
		Canada	1981			Austria	1981	Spain	1992
		France	1992			Austria	1974	Sweden	1970
		Germany	1974			Austria	1980	Sweden	1975
		Germany	1980			Austria	1992	Sweden	1980
		Italy	1992			Austria	2001	Switzerland	1974
		Spain	1980			Canada	1990	Switzerland	1981
		Sweden	1990			France	1974	Switzerland	1990
		United Kingdom	1974			France	1980	Switzerland	2001
		United Kingdom	1979			France	1982	United States	1953
		United Kingdom	1990			France	2002	United States	1957
		United States	1981			Germany	1966	United States	1960
		United States	1990			Germany	1991	United States	1969
						Germany	2001	United States	1973
						Italy	1970	United States	1980
						Italy	1974	United States	2001
Emerging Market Economies									
Low Inflation	Algeria	1985	Mexico	2000	Chile	1997			
	Algeria	1992	Morocco	1980	Morocco	1991			
	Argentina	1994	Morocco	1982	Morocco	1994			
	Brazil	1997	Morocco	1986	Morocco	1996			
	Colombia	1995	Panama	1982	Philippines	1990			
	Côte d'Ivoire	1982	Panama	1986	Tunisia	1981			
	Côte d'Ivoire	1986	Peru	1997	Tunisia	1985			
	Côte d'Ivoire	1991	Philippines	1997	Venezuela	1997			
	Côte d'Ivoire	1998	South Africa	1981					
	Côte d'Ivoire	2001	South Africa	1984					
	Korea	1996	South Africa	1989					
	Malaysia	1984	Thailand	1996					
	Malaysia	1997							
High Inflation	Algeria	1989	Mexico	1994	Argentina	1984			
	Argentina	1980	Nigeria	1980	Dominican Rep	1989			
	Argentina	1987	Peru	1981	Ecuador	1986			
	Argentina	1998	Peru	1987	Lebanon	1991			
	Brazil	1980	Philippines	1983	Mexico	1985			
	Brazil	1987	Russia	1997	Uruguay	1994			
	Brazil	1991	Turkey	1993					
	Bulgaria	1995	Turkey	1997					
	Chile	1981	Turkey	2000					
	Dominican Rep.	2000	Uruguay	1981					
	Ecuador	1981	Uruguay	1998					
	Ecuador	1998	Venezuela	1980					
	El Salvador	1980	Venezuela	1988					
	Indonesia	1997	Venezuela	1992					
	Lebanon	1988	Venezuela	1995					
	Mexico	1981	Venezuela	2001					

B.2 Proofs

B.2.1 Proof of Proposition 2

Around a solution of the firms' problem (K^*, L^*) in which $\lambda > 0$, the complementary slackness condition (2.7) implies that (2.4) holds with equality: $(Z - \theta K - WL) = 0$. First, consider the case with $\theta > 0$. This means that $L = (Z/W - (\theta/W)K) \equiv L(K)$, with $\partial L(K)/\partial K < 0$. Combining conditions (2.5) and (2.6), we obtain

$$S(K, A) \equiv A \left(F_L(K, L(K)) - \frac{W}{\theta} F_K(K, L(K)) \right) + W \left(\frac{1}{\theta} - 1 \right) = 0.$$

By the implicit function theorem,

$$\frac{\partial K}{\partial A} = \frac{\partial S(K, A)/\partial K}{\partial S(K, A)/\partial A}.$$

If $\theta < 1$, $\partial K/\partial A > 0$ since

$$\frac{\partial S(K, A)}{\partial A} = \left(F_L(K, L(K)) - \frac{W}{\theta} F_K(K, L(K)) \right) = \frac{W}{A} \left(1 - \frac{1}{\theta} \right) < 0$$

and, by linear homogeneity and diminishing return on each factor,

$$F_{LK} + F_{LL} \frac{\partial L(K)}{\partial K} - \frac{W}{\theta} F_{KK} + -\frac{W}{\theta} F_{KL} \frac{\partial L(K)}{\partial K} > 0.$$

Finally (2.4) holding with equality implies that $\partial L/\partial A < 0$ and thus $\partial L/\partial A < \partial K/\partial A$. Second, consider the case with $\theta = 0$. Equation (2.4) holding with equality implies that $L = Z/W$ and thus $\partial L/\partial A = 0$. Equation (2.5) implies that $AF_K(K, Z/W) = 1$, and the implicit function theorem implies that $\partial K/\partial A > 0$ and thus $\partial L/\partial A < \partial K/\partial A$.

B.3 A Quantitative Exercise of the Analytical Framework

In this section, we perform a simple quantitative exercise to show that the analytical framework presented in Section 2.5 can rationalize actual jobless recovery episodes. In

particular, we calibrate the model and compare its predictions with actual data from the US Great Recession. We begin by assuming the technology is Cobb–Douglas:

$$F(K, L) = K^\alpha L^{1-\alpha}. \quad (\text{B.1})$$

We assume, for simplicity, that $\theta = 0$, corresponding to the case in which K is its own collateral. Furthermore, we assume that real wages are constant ($\Delta w_t = 0$ for every t). This assumption is consistent with US data for the Great Recession ([Shimer, 2012](#)). We now solve the model for the case in which the credit constraint is binding, and thus Equation (2.4) holds with equality. Thus, $wL = Z$ and profits can be expressed as

$$AZ^{1-\alpha} = K^\alpha - (K + Z). \quad (\text{B.2})$$

The first-order condition with respect to capital implies

$$K = \alpha^{\frac{1}{1-\alpha}} Z A^{\frac{1}{1-\alpha}}. \quad (\text{B.3})$$

Hence, assuming discrete time and denoting for any variable X , $\Delta x_t = \log X_t - \log X_{t-1}$, we get

$$\Delta l_t = \Delta z_t \quad (\text{B.4})$$

$$\Delta k_t = \frac{1}{1-\alpha} \Delta a_t + \Delta z_t. \quad (\text{B.5})$$

Our aim is to compare the model's prediction for L and K with US data during the Great Recession. The time unit is set equal to a quarter. We obtain data for L , K , A and utilization from [Fernald \(2012\)](#). For L , we use hours worked; for A , we use total factor productivity adjusted by utilization. We estimate Z , as the process consistent with the model that reproduces the actual behavior of Y , that is, using (B.4) and (B.5), $\Delta z_t = \Delta y_t - 1/(1-\alpha)\Delta a_t$. Following the estimate of [Fernald \(2012\)](#) for 2007, we set $\alpha = 0.35$ the year prior to the crisis. Results are presented in Figure B.1. Panels a) and b) show the behavior of A and Z , the model's inputs. It can

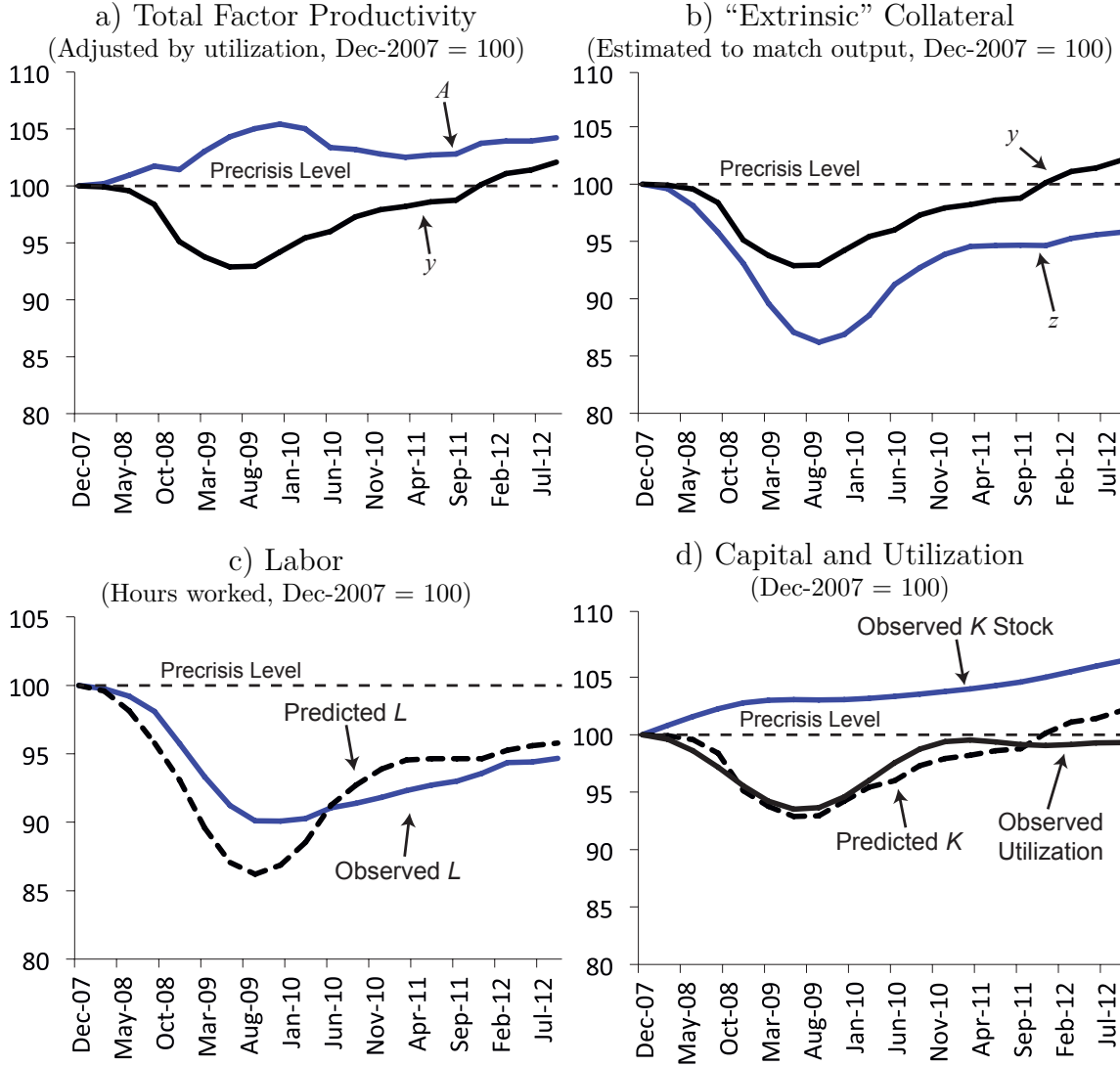


FIGURE B.1: Model Predictions: US Great Recession

Data Source: TFP adjusted by utilization, observed labor, capital and utilization: [Fernald \(2012\)](#)

be seen that A increases throughout the episode, while the estimated Z displays a sharp contraction, consistent with the behavior of output. Panels c) and d) depict the behavior of L and K . It can be observed that employment behavior predicted by the model tracks closely the actual path of employment. In particular, the model predicts a jobless recovery similar to the one observed in the data.

Appendix C

Appendix for Chapter 3

C.1 Omitted Constraints in Ramsey Problems

In Section 3.1, to characterize the allocation under the different exchange-rate regimes, I follow the strategy of setting up the Ramsey problem dropping constraints; this Appendix shows that the omitted constraints are satisfied.

C.1.1 Optimal Exchange-Rate Policy

This section shows that any $\{d_{t+1}, c_t^T, h_t\}$ that satisfy (3.5), (3.12), (3.17), and (3.18) also satisfy (GE). Pick $c_t^N = F(h_t)$ to satisfy (3.15). Pick $p_t = \left(\frac{1-a}{a}\right) \left(\frac{c_t^T}{c_t^N}\right)^{\frac{1}{\xi}}$ to satisfy (3.8). Pick $\mu_t = 0$. This makes (3.9) hold. Pick $\lambda_t = U_c(c_t^T, c_t^N) A_T(c_t^T, c_t^N)$ to satisfy (3.7). Choose $\tau_t^d = 1 - R_t \beta \frac{E_t \lambda_{t+1} + \mu_t}{\lambda_t}$ to satisfy (3.6). Choose T_t to satisfy (3.14) as: $T_t = \tau_t^d d_{t+1} R_t^{-1}$. Pick $w_t = \left(\frac{1-a}{a}\right) (c_t^T)^{\frac{1}{\xi}} F(h_t)^{-\frac{1}{\xi}} F'(h_t)$ to satisfy (3.16). Given w_{t-1} , pick ϵ_t to satisfy (3.11) with equality: $\epsilon_t = \gamma \frac{w_{t-1}}{w_t}$. Finally, since (3.11) holds with equality, (3.13) always holds: $\left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t}\right) (\bar{h} - h_t) = 0$.

C.1.2 Full-Employment Exchange-Rate Policy

This section shows that any $\{d_{t+1}, c_t^T, h_t\}$ that satisfy (3.5), (3.17), (3.18), and (3.20), also satisfy (GE) and (3.20). Pick $c_t^N = F(\bar{h})$ to satisfy (3.15). Pick $p_t = \left(\frac{1-a}{a}\right) \left(\frac{c_t^T}{c_t^N}\right)^{\frac{1}{\xi}}$ to satisfy (3.8). Pick $\mu_t = 0$. This makes (3.9) hold. Pick $\lambda_t = U_c(c_t^T, c_t^N) A_T(c_t^T, c_t^N)$ to satisfy (3.7). Choose $\tau_t^d = 1 - R_t \beta \frac{E_t \lambda_{t+1} + \mu_t}{\lambda_t}$ to satisfy (3.6). Choose T_t to satisfy (3.14) as: $T_t = \tau_t^d d_{t+1} R_t^{-1}$. Pick $w_t = \left(\frac{1-a}{a}\right) (c_t^T)^{\frac{1}{\xi}} F(\bar{h})^{-\frac{1}{\xi}} F'(\bar{h})$

to satisfy (3.16). Given w_{t-1} , pick ϵ_t to satisfy (3.11) with equality: $\epsilon_t = \gamma \frac{w_{t-1}}{w_t}$. Finally, by (3.20) and (3.12), (3.13) always holds: $\left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t}\right) (\bar{h} - h_t) = 0$.

C.1.3 Fixed Exchange-Rate Policy

This section shows that any $\{d_{t+1}, c_t^T, h_t, w_t, \epsilon_t\}$ that satisfy (3.5), (3.11)–(3.13), (3.16)–(3.18), and (3.22), also satisfy (GE) and (3.22). Pick $c_t^N = F(h_t)$ to satisfy (3.15). Pick $p_t = \left(\frac{1-a}{a}\right) \left(\frac{c_t^T}{c_t^N}\right)^{\frac{1}{\xi}}$ to satisfy (3.8). Pick $\mu_t = 0$. This makes (3.9) holds. Pick $\lambda_t = U_c(c_t^T, c_t^N) A_T(c_t^T, c_t^N)$ to satisfy (3.7). Choose $\tau_t^d = 1 - R_t \beta \frac{E_t \lambda_{t+1} + \mu_t}{\lambda_t}$ to satisfy (3.6). Choose T_t to satisfy (3.14) as: $T_t = \tau_t^d d_{t+1} R_t^{-1}$.

C.2 Proofs

C.2.1 Proof of Proposition 3

The Ramsey problem of optimal exchange-rate policy under an optimal capital-control tax is to maximize (3.1) with respect to $\{d_{t+1}, c_t^T, h_t\}$, subject to (3.5), (3.12), (3.17) and (3.18). The Lagrangean of the Ramsey problem is then given by

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{aligned} & U(A(c_t^T, F(h_t))) + \\ & + \phi_t^F \left[\frac{d_{t+1}}{R_t} - d_t + y_t^T - c_t^T \right] + \\ & + \phi_t^\mu \left[\kappa \left(y_t^T + \left(\frac{1-a}{a} \right) (c_t^T)^{\frac{1}{\xi}} F(h_t)^{1-\frac{1}{\xi}} \right) - d_{t+1} \right] + \\ & + \phi_t^\eta [d^N - d_{t+1}] + \\ & + \phi_t^W [\bar{h} - h_t] \end{aligned} \right\},$$

where ϕ_t^F , ϕ_t^μ , ϕ_t^η , and ϕ_t^W are Lagrange multipliers.

The optimality conditions associated with this problem (provided $d_{t+1} < d^N$) are

(3.12), (3.17), (3.18), the first-order conditions

$$\frac{\phi_t^F}{R_t} = \beta \mathbb{E}_t \phi_{t+1}^F + \phi_t^\mu, \quad (\text{C.1})$$

$$\phi_t^F = U_c A_T (c_t^T, F(h_t)) + \phi_t^\mu \kappa \left(\frac{1}{\xi} \right) \left(\frac{1-a}{a} \right) \left(\frac{c_t^T}{F(h_t)} \right)^{\frac{1}{\xi}-1}, \quad (\text{C.2})$$

$$\phi_t^W = F'(h_t) \left[U_c A_N (c_t^T, F(h_t)) + \phi_t^\mu \left(\frac{\xi-1}{\xi} \right) \kappa \left(\frac{1-a}{a} \right) \left(\frac{c_t^T}{F(h_t)} \right)^{\frac{1}{\xi}} \right], \quad (\text{C.3})$$

and the complementary slackness conditions

$$\phi_t^\mu \geq 0; \quad \phi_t^\mu \left[\kappa \left(y_t^T + \left(\frac{1-a}{a} \right) (c_t^T)^{\frac{1}{\xi}} F(h_t)^{1-\frac{1}{\xi}} \right) - d_{t+1} \right] = 0, \quad (\text{C.4})$$

$$\phi_t^W \geq 0; \quad \phi_t^W [\bar{h} - h_t] = 0. \quad (\text{C.5})$$

First, consider the case with $\xi < 1$. Assume, contrary to the statement of the proposition, that under the optimal exchange-rate policy, at some date t , $h_t < \bar{h}$ and $d_{t+1} < \bar{d}(h_t, c_t^T, y_t^T)$. By (C.5) it follows that $\phi_t^W = 0$. By (C.3), and since $c_t^T > 0$, $h_t > 0$, $F'(h_t) > 0$, $U_c A_N (c_t^T, F(h_t)) > 0$, and $\left(\frac{\xi-1}{\xi} \right) < 0$, this implies that $\phi_t^\mu > 0$, which contradicts (C.4), which requires $\phi_t^\mu (\bar{d}(h_t, c_t^T, y_t^T) - d_{t+1}) = 0$.

Second, consider the case with $\xi \geq 1$. Assume, contrary to the statement of the proposition, that under the optimal exchange-rate policy, at some date t , $h_t < \bar{h}$. By (C.3), and since $c_t^T > 0$, $h_t > 0$, $F'(h_t) > 0$, $U_c A_N (c_t^T, F(h_t)) > 0$, $\left(\frac{\xi-1}{\xi} \right) \geq 0$ and $\phi_t^\mu \geq 0$, this implies that $\phi_t^W > 0$, which contradicts (C.5), which requires $\phi_t^W (\bar{h} - h_t) = 0$.

C.2.2 Proof of Proposition 4

Given the initial state (s_t^X, d_t) and a debt level d_{t+1}^* with associated tradable consumption $c_t^{T*} = (d_{t+1}^* R_t^{-1} - d_t + y_t^T)$, pick h_t^* such that $d_{t+1}^* = \bar{d}(h_t^*, c_t^{T*}, y_t^T)$. This implies setting h_t^* as:

$$h_t^* = F^{-1} \left(\left(\left(\frac{1-a}{a} \right) (d_{t+1}^* R_t^{-1} - d_t + y_t^T)^{\frac{1}{\xi}} (d_{t+1}^* \kappa^{-1} - y_t^T)^{-1} \right)^{\frac{\xi}{1-\xi}} \right).$$

By assumption $d_{t+1}^* > \bar{d}(\bar{h}, c_t^{T*}, y_t^T)$. Since $\xi < 1$, it follows that

$\bar{h} > F^{-1} \left(\left(\left(\frac{1-a}{a} \right) (d_{t+1}^* R_t^{-1} - d_t + y_t^T)^{\frac{1}{\xi}} (d_{t+1}^* \kappa^{-1} - y_t^T)^{-1} \right)^{\frac{\xi}{1-\xi}} \right)$, and thus $h_t^* < \bar{h}$.

Finally, since $(d_{t+1}^* R_t^{-1} - d_t + y_t^T)^{\frac{1}{\xi}} > 0$, and $F(\bar{h}) > 0$, by the assumption that $d_{t+1}^* > \bar{d}(\bar{h}, c_t^*, y_t^T)$, it also follows that $d_{t+1}^* > \kappa y_t^T$, and thus $h_t^* > 0$.

C.2.3 Proof of Remark 1

As shown in Section 7.2.1, the optimality conditions associated with the problem of optimal exchange-rate policy with capital-control taxes (Definition 1) are (3.12), (3.17), (3.18), (C.1), (C.2) (C.3), (C.4) and (C.5). In an allocation in which at time t , $h_t < \bar{h}$, by (C.5) it follows that $\phi_t^W = 0$. Replacing in (C.3), (3.24) is obtained.

C.3 Data Appendix

1. Sectoral data, Argentina: Constructed using data on value added from agriculture, manufacturing, and services from the WDI dataset. The tradable sector was defined as the sum of agriculture and manufacturing sectors. The nontradable sector was defined as services. Data on relative prices was constructed using current and constant value added on each sector.
2. External debt, Argentina: Measured using net foreign assets, obtained from Lane and Milesi-Ferreti (2007) dataset.
3. National accounts, Argentina: Output, consumption, and net exports, obtained from WEO dataset.
4. Interest rates: For Argentina, since 1994, the country interest rate on external debt was measured as the sum of country EMBI spreads and the US Treasury-Bill rate, obtained, respectively, from Datastream and the Federal Reserve of Saint Louis datasets. The series were extended back to 1983 using Neumeyer and Perri (2005) dataset, which uses a measure similar to the one considered here. The risk-free

rate was measured with the Treasury-Bill rate. The interest rate series is then deflated with a measure of expected dollar inflation. In particular, R_t is measured as $R_t = (1 + i_t) E_t \left(\frac{1}{1 + \pi_{t+1}^*} \right)$, where i_t denotes the interest rate on Argentinean external debt in US dollars, and π_{t+1}^* denotes US CPI. $E_t \left(\frac{1}{1 + \pi_{t+1}^*} \right)$ is obtained as the one-step-ahead forecast of an estimated AR(1). US CPI data was obtained from the Federal Reserve of Saint Louis dataset. For EMs, spreads (Figure 4) were measured with the EMBI, obtained from Datastream.

5. Unemployment rate, Argentina: Obtained from INDEC. Since 2003, excludes government social plan “Jefas y Jefes ” .
6. Nominal wage, Argentina: Obtained from [Calvo, Coricelli and Ottonello \(2012\)](#) dataset.
7. Balance of payments, Argentina: Current account and factor services obtained from IFS dataset.
8. Measured TFP, Argentina: Obtained from [Aravena and Fuentes \(2013\)](#) dataset.
9. Nominal exchange rates and international reserves, EMs: Obtained from IFS dataset.

C.4 Recursive Formulation of the Optimal Exchange-Rate Policy without Capital-Control Taxes

This section shows how to obtain a recursive formulation of the problem of optimal exchange-rate policy without capital-control taxes, using the recursive saddle-point method developed in [Marcet and Marimon \(2011\)](#). Without capital-control taxes, the general equilibrium dynamics are given by stochastic processes

$\{c_t^N, c_t^T, h_t, p_t, w_t, d_{t+1}, \lambda_t, \mu_t\}_{t=0}^\infty$ satisfying the set of equations (GE'): $\{(3.5)–(3.9), (3.11)–(3.13), (3.15)–(3.18)\}$ given an exchange-rate policy $\{\epsilon_t\}_{t=0}^\infty$, initial conditions w_{-1} and d_0 , and exogenous stochastic processes $\{y_t^T, R_t\}_{t=0}^\infty$.

To characterize the allocation under the optimal exchange-rate policy, without optimal capital-control taxes, and under commitment, I begin by setting up the Ramsey problem and dropping constraints (3.8), (3.11), (3.13), and (3.15)–(3.16). Any $\{d_{t+1}, c_t^T, h_t, \lambda_t, \mu_t\}$ that satisfy (3.5)–(3.7), (3.9), (3.12), (3.17), and (3.18), also satisfy (GE'). To see this, pick $c_t^N = F(h_t)$ to satisfy (3.15). Pick $p_t = \left(\frac{1-a}{a}\right) \left(\frac{c_t^T}{c_t^N}\right)^{\frac{1}{\xi}}$ to satisfy (3.8). Pick $w_t = \left(\frac{1-a}{a}\right) (c_t^T)^{\frac{1}{\xi}} F(h_t)^{-\frac{1}{\xi}} F'(h_t)$ to satisfy (3.16). Given w_{t-1} , pick ϵ_t to satisfy (3.11) with equality: $\epsilon_t = \gamma \frac{w_{t-1}}{w_t}$. Finally, since (3.11) holds with equality, (3.13) always holds: $\left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t}\right) (\bar{h} - h_t) = 0$. The Ramsey problem is then given by

$$\begin{aligned} \max_{\{d_{t+1}, c_t^T, h_t, \lambda_t, \mu_t\}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, F(h_t))) \\ \text{s.t.} \quad & d_{t+1} \leq d^N, \\ & \lambda_t R_t^{-1} = \beta \mathbb{E}_t \lambda_{t+1} + \mu_t, \\ & U_c(c_t^T, c_t^N) A_T(c_t^T, c_t^N) = \lambda_t, \\ & \mu_t \geq 0, \\ & \mu_t (\kappa (y_t^T + p_t F(h_t)) - d_{t+1}) = 0, \\ & h_t \leq \bar{h}, \\ & d_{t+1} R_t^{-1} = d_t + c_t^T - y_t^T, \\ & d_{t+1} \leq \kappa \left(y_t^T + \left(\frac{1-a}{a} \right) (c_t^T)^{\frac{1}{\xi}} F(h_t)^{\frac{\xi-1}{\xi}} \right). \end{aligned} \tag{C.6}$$

To obtain the recursive formulation using the method of [Marcet and Marimon \(2011\)](#), the following steps are followed (as in [Adam and Billi, 2005](#)). First, set the Lagrangean of problem (C.6), denoting ϕ_t^D the Lagrange multiplier associated with the forward looking constraint (3.6). Since equation (3.6) is forward-looking,

some terms in the Lagrangean involve period t Lagrange multipliers multiplied by period $t + 1$ controls (e.g., $\phi_t^D \mathbb{E}_t \lambda_{t+1}$). Second, in these terms, relabel the Lagrange multipliers multiplied by period $t + 1$ controls as $\tilde{\phi}_{t+1}^D$. This relabeling defines a “transition” equation: $\phi_t^D = \tilde{\phi}_{t+1}^D$. Third, define the period objective function,

$$\begin{aligned} H \left(c_t^T, h_t, \mu_t, \tilde{\phi}_t^D, \phi_t^D \right) \equiv & U \left(A \left(c_t^T, F(h_t) \right) \right) - \\ & - U_c \left(c_t^T, F(h_t) \right) A_T \left(c_t^T, F(h_t) \right) \left(\frac{\phi_t^D}{R_t} - \tilde{\phi}_t^D \right) + \phi_t^D \mu_t. \end{aligned} \quad (\text{C.7})$$

Fourth, using the equivalence results shown in [Marcet and Marimon \(2011\)](#), re-express the problem (C.6) as an infinite-horizon saddle-point problem:

$$\begin{aligned} \min_{\phi_t^D} \max_{\{d_{t+1}, c_t^T, h_t, \mu_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t H \left(c_t^T, h_t, \mu_t, \tilde{\phi}_t^D, \phi_t^D \right) \quad & (\text{C.8}) \\ \text{s.t. } \tilde{\phi}_{t+1}^D = \phi_t^D & \\ d_{t+1} \leq d^N, & \\ \mu_t \geq 0, & \\ \mu_t \left(\kappa \left(y_t^T + p_t F(h_t) \right) - d_{t+1} \right) = 0, & \\ h_t \leq \bar{h}, & \\ d_{t+1} R_t^{-1} = d_t + c_t^T - y_t^T, & \\ d_{t+1} \leq \kappa \left(y_t^T + \left(\frac{1-a}{a} \right) (c_t^T)^{\frac{1}{\xi}} F(h_t)^{\frac{\xi-1}{\xi}} \right), & \\ \tilde{\phi}_0 = 0. & \end{aligned}$$

Finally, rewrite the infinite-horizon saddle-point problem (C.8) in recursive form:

$$W^{OP} \left(s^X, d, \tilde{\phi}^D \right) = \min_{\tilde{\phi}^{D'}} \max_{d', c^T, h, \mu} \left[H \left(c^T, h, \mu, \tilde{\phi}^D, \tilde{\phi}^{D'} \right) + \beta \mathbb{E}_{s^{X'}} W^{OP} \left(s^{X'}, d', \tilde{\phi}^{D'} \right) \right] \quad (\text{C.9})$$

$$\begin{aligned} s.t. \quad & \frac{d'}{R} = d + c^T - y^T, \\ & d' \leq \kappa \left(y^T + \left(\frac{1-a}{a} \right) (c^T)^{\frac{1}{\xi}} F(h)^{\frac{\xi-1}{\xi}} \right), \\ & d' \leq d^N, \\ & h \leq \bar{h}, \\ & \mu \geq 0, \\ & \mu \left(\kappa \left(y^T + \left(\frac{1-a}{a} \right) (c^T)^{\frac{1}{\xi}} F(h)^{\frac{\xi-1}{\xi}} \right) - d' \right) = 0, \end{aligned}$$

where time subscripts for variables dated in period t have been dropped, and a prime is used to indicate variables dated in period $t + 1$.